

# Mean-Adjusted Pattern-Dependent Noise Prediction for Perpendicular Recording Channels with Nonlinear Transition Shift

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**Abstract**—In high density perpendicular magnetic recording channels, nonlinear transition shift (NLTS) is one of the distortions that can degrade the system performance. Write precompensation is a standard method used to combat the negative effect of NLTS. In this paper, we propose a modified pattern-dependent noise predictive (PDNP) detection algorithm for use on channels with electronics noise, transition jitter noise, and NLTS. We show that this detector can offer significant improvement in bit-error-rate (BER) compared to conventional Viterbi and PDNP detectors. Moreover, the system performance can be further improved by combining the new detector with a simple write precompensation scheme.

**Index Terms**—NLTS, pattern-dependent noise prediction, jitter noise, perpendicular recording

## I. INTRODUCTION

As the areal density in a perpendicular recording system increases, nonlinear effects in the recording and readback process can have a more significant impact on overall performance. Nonlinear transition shift (NLTS) is an important example of such effects. NLTS occurs during the write process, and it is caused by magnetization effects due to previously recorded transitions. The extent of the transition shift depends on the head and media parameters, as well as on the preceding data pattern. Write precompensation is a method that is widely used to partially counteract the distortion induced by NLTS.

In this paper, we propose a new detection method designed to further reduce the performance degradation resulting from NLTS effects. We show that the new detector performs significantly better than a conventional Viterbi detector and a pattern-dependent noise prediction (PDNP) detector, while its computational complexity is comparable to that of a PDNP detector. In contrast to write precompensation, which generally requires empirical optimization of the precompensation levels, the new detection algorithm incorporates parameters that reflect the channel nonlinearity and noise statistics. These parameters can be determined either adaptively or through the use of training sequences.

Several equalization and detection techniques have previously been proposed to reduce the effects of nonlinear distortion and media noise in the magnetic recording readback

channel. A Volterra equalizer design method was proposed in [1] to combat channel nonlinearities. Detector design has generally concentrated on techniques that mitigate the effects of nonlinear media noise and noise correlation. In particular, the conventional Viterbi detector has been modified in various ways. For example, in [2], [3], partial local feedback noise prediction was used to reduce the impact of correlated noise and media noise. Kavčić and Moura [4] derived a maximum-likelihood sequence detector (MLSD) for an intersymbol-interference (ISI) channel with data-dependent finite-memory Gauss-Markov noise and applied the detector to an autoregressive (AR) model for the magnetic recording channel. The detector incorporates pattern-dependent noise prediction filters. Moon and Park [5] examined various sub-optimal pattern-dependent noise prediction (PDNP) detectors that offer a trade-off between performance and implementation complexity in the presence of media noise. Zayed and Carley [6] and Sun et al. [7] both proposed a modified Viterbi detector with a data-dependent offset in the branch metric calculation, intended to deal with both nonlinearities and media noise.

In this paper, we address the data-dependent nature of NLTS and derive a modified PDNP detector for perpendicular recording channels with additive Gaussian noise, transition jitter noise, and NLTS. Computer simulations show that the new detector, which we refer to as the mean-adjusted PDNP (MA-PDNP) detector, improves the bit-error-rate (BER) performance when compared to the conventional Viterbi detector and the PDNP detector. In our simulations, we calculate the NLTS according to the model proposed by Bertram et al. [8], [9]. We also show that the MA-PDNP detector can be combined with write precompensation schemes to achieve further performance improvement.

The paper is organized as follows. In Section II, the channel model is introduced. Section III describes the structure of the MA-PDNP detector. Section IV gives the simulation results and comparisons between the MA-PDNP detector, Viterbi detector, and PDNP detector. The performance of the MA-PDNP detector combined with a dibit precompensation scheme is also presented. Section V concludes the paper.

## II. CHANNEL MODEL

We consider a channel model with AWGN, jitter noise, and NLTS, described in [10]. Let the channel transition response

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$$s(t) = V_{\max} \operatorname{erf} \left( \frac{0.954t}{T_{50}} \right), \quad (1)$$

where  $\operatorname{erf}(\cdot)$  is the error function and  $T_{50}$  is the width of the transition response at half of its maximum amplitude.

Let  $\{x_i\}$  be the binary input data sequence to the channel, where  $x_i \in \{-1, +1\}$ . The corresponding transition sequence is given by  $d_i = \frac{x_i - x_{i-1}}{2}$ , so  $d_i \in \{-1, 0, +1\}$ . The channel output  $z(t)$  can be written as

$$z(t) = \sum_i d_i s(t + \delta_i + a_i - iB) + n_W(t). \quad (2)$$

Here,  $\delta_i$  is the net shift of the transition  $d_i$  with respect to its nominal location in the recording medium,  $a_i$  is the random position jitter for transition  $d_i$ ,  $B$  is the channel bit spacing (as well as the sampling period), and  $n_W(t)$  is the electronics noise. For each transition, the jitter value  $a_i$  is assumed to be a zero-mean Gaussian random variable with variance  $\sigma_j^2$ . The jitter values for recorded transitions are mutually independent. The electronics noise  $n_W(t)$  is modeled as a zero-mean, AWGN process. The variance of the sampled AWGN  $n_W(iB)$  is denoted by  $\sigma_W^2$ . We define the signal-to-AWGN ratio to be  $SNR_W = 10 \log_{10}(V_{\max}^2/\sigma_W^2)$ .

The net transition shift is given by  $\delta_i = \tau_i + \Delta_i$ , where  $\tau_i$  is the NLTS induced by previously recorded transitions, and  $\Delta_i$  is the precompensation applied to the transition  $d_i$ . The NLTS value  $\tau_i$ , determined according to the NLTS model in [8], [9], is a function of  $\Delta_i$  and the locations of the previously written transitions (including any precompensation applied to them) and depends upon the parameters of the recording medium and head.

In order to reduce computational complexity in our simulations, we approximate the channel output by truncating the Taylor series expansion of the transition response. Specifically, we use an order-2 channel approximation, incorporating both first- and second-derivative terms, as given by

$$z(t) \approx \sum_i d_i s(t - iB) + \sum_i d_i (\delta_i + a_i) s'(t - iB) + \sum_i d_i \frac{(\delta_i + a_i)^2}{2} s''(t - iB) + n_W(t), \quad (3)$$

where  $s'(t)$  and  $s''(t)$  are the first and second derivatives of the transition response  $s(t)$ , respectively.

The system diagram is shown in Fig. 1. During the readback process, the channel output is sampled and then passed through the equalizer before it enters the detector. The discrete-time signal at the detector input can thus be written as

$$r_k = \sum_i x_i \tilde{h}_{k-i} + \sum_i d_i (\delta_i + a_i) \tilde{s}'_{k-i} + \sum_i d_i \frac{(\delta_i + a_i)^2}{2} \tilde{s}''_{k-i} + w_k, \quad (4)$$

where  $\tilde{h}_j$ ,  $\tilde{s}'_j$  and  $\tilde{s}''_j$  denote the convolution of the FIR equalizer taps with the samples of the channel dibit response, the first derivative of the transition response, and the second derivative of the transition response, respectively. The equalized sample of the AWGN at time  $k$  is denoted  $w_k$ . With a

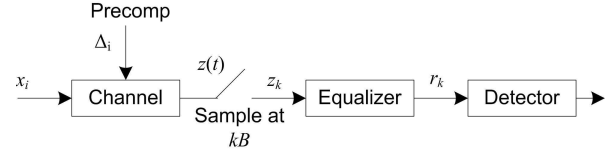


Fig. 1. System diagram

target equalizer response  $g(D) = g_0 + g_1 D + \dots + g_J D^J$ , the equalizer output can be written as

$$r_k = \sum_{i=0}^J g_i x_{k-i} + n_k, \quad (5)$$

where

$$n_k = \sum_i d_i (\delta_i + a_i) \tilde{s}'_{k-i} + \sum_i d_i \frac{(\delta_i + a_i)^2}{2} \tilde{s}''_{k-i} + w_k + v_k. \quad (6)$$

The noise  $n_k$ , which has non-zero mean and a non-Gaussian density, includes contributions from the NLTS, transition jitter noise, equalized AWGN, and misequalization error  $v_k$ .

### III. MEAN-ADJUSTED PATTERN-DEPENDENT NOISE PREDICTION

In this section, we will first give some background on the PDNP detector and then introduce the MA-PDNP detector.

#### A. PDNP detector

The conventional Viterbi algorithm with squared-Euclidean metric is an MLSD only if the noise term  $n_k$  in (5) is sampled AWGN. The number of trellis states in such a Viterbi detector is  $2^J$ . For a channel with zero-mean, data-dependent, finite-memory Gauss-Markov noise, the MLSD was derived in [4]. We refer to this as the pattern-dependent noise-prediction (PDNP) detector. The branch metric corresponding to a trellis branch  $e_k$  at time  $k$  can be expressed as

$$\mathcal{M}(e_k) = \log(\sqrt{2\pi\sigma^2(e_k)}) + \frac{1}{2\sigma^2(e_k)} \left[ \sum_{i=0}^L p_i(e_k) (r_{k-i} - y_{k-i}) \right]^2, \quad (7)$$

where  $L$  is the memory length of the Markovian noise process, and  $y_j(e_k)$ ,  $j = k-L, \dots, k$  denote the target equalizer output samples. The noise variance  $\sigma^2(e_k)$  and the noise prediction coefficients  $p_j(e_k)$ ,  $j = 1, \dots, L$  are data-dependent and can be calculated from the noise covariance matrix. Denoting the covariance matrix for  $\mathbf{n} = (n_{k-L}, \dots, n_k)$  by  $\mathbf{R}_L$  and for  $\mathbf{n}' = (n_{k-L}, \dots, n_{k-1})$  by  $\mathbf{R}_{L-1}$ , we can write

$$\mathbf{R}_L = \begin{pmatrix} \mathbf{R}_{L-1} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^T & \gamma_{kk} \end{pmatrix}, \quad (8)$$

where  $\boldsymbol{\gamma}$  is a column vector and  $\gamma_{kk} = E\{n_k^2\}$ . The noise variance and prediction coefficients are then given by

$$\sigma^2(e_k) = \gamma_{kk} - \boldsymbol{\gamma}^T \mathbf{R}_{L-1}^{-1} \boldsymbol{\gamma}, \quad (9)$$

$$\mathbf{p}(e_k) = -\mathbf{R}_{L-1}^{-1} \boldsymbol{\gamma}, \quad (10)$$

where  $\mathbf{p}(e_k) = (p_L(e_k), \dots, p_1(e_k))^T$  and  $p_0(e_k) = 1$ .

The PDNP trellis complexity is increased by the memory length of the Markovian noise as well as the span of its data dependence. For example, suppose that the signal-dependent noise  $n_k$  depends on the data values in positions  $k - C$  to  $k + D$ . Then, the trellis state  $s_k$  at time  $k$  is defined by  $s_k = (x_{k-L-\max\{C,J\}+1}, \dots, x_{k+D})$ , and the number of states is therefore  $2^{L+D+\max\{C,J\}}$ .

The noise in a magnetic recording channel is not finite-memory Gauss-Markov, so the PDNP detector is, strictly speaking, not optimal, although it achieves near-optimal performance [5]. Several methods have been proposed to reduce the complexity of the PDNP detector, yielding a variety of simpler, yet still effective detectors. Strategies include reducing the prediction filter length, limiting the required number of predictors by shortening the data-dependence length, and eliminating trellis states by using feedback of tentative decisions.

### B. MA-PDNP detector

As we discussed in Section II, the noise  $n_k$  in (6) is non-Gaussian with non-zero mean. Along with the jitter noise, the NLTS affects the noise mean and variance. However, unlike the jitter noise, the NLTS is a deterministic, data-dependent nonlinear effect. To account for the NLTS, we propose a mean-adjusted PDNP (MA-PDNP) detector in which the branch metric for a branch  $e_k$  is written as:

$$\begin{aligned} \mathcal{M}(e_k) = & \log(\sqrt{2\pi\sigma^2(e_k)}) \\ & + \frac{1}{2\sigma^2(e_k)} \left[ \sum_{i=0}^L p_i(e_k)(r_{k-i} - y_{k-i} - m_i(e_k)) \right]^2, \end{aligned} \quad (11)$$

where  $m_i(e_k)$  represents the data-dependent mean of the noise  $n_{k-i}$ .

The methods used to reduce the complexity of the PDNP detector can also be applied to the MA-PDNP detector. More precisely, we can rewrite the branch metric in (11) as

$$\begin{aligned} \mathcal{M}(\mathbf{x}_k) = & \log(\sqrt{2\pi\sigma^2(\mathbf{x}_k)}) \\ & + \frac{1}{2\sigma^2(\mathbf{x}_k)} \left[ \sum_{i=0}^{L_y} p_i(\mathbf{x}_k)(r_{k-i} - y_{k-i} - m_i(\mathbf{x}_k)) \right]^2, \end{aligned} \quad (12)$$

where  $\mathbf{x}_k$  represents the data pattern from which the noise variance and the prediction filter coefficients corresponding to time  $k$  are determined. Since the noise is not a finite-memory Markovian process, we can not assign a value for the memory length  $L$ . Instead, we specify a value  $L_y$  to represent the length of the noise prediction filters, and we denote the span of the data-dependence by  $L_x + 1$ . The number of trellis states is given by  $2^M$ . When  $M < L_x$ , the data pattern  $\mathbf{x}_k$  includes tentative decisions from the survivor path ending at the initial state of the branch  $e_k$ .

The computational complexity of the MA-PDNP detector depends upon the parameters  $M$  and  $L_y$ , while  $L_x$  determines the memory size required to store the pattern-dependent mean, variance and noise prediction filter coefficients. Thus, the

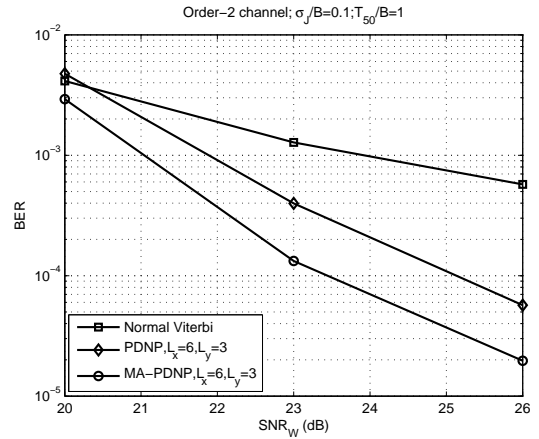


Fig. 2. Comparison between Viterbi, PDNP, and MA-PDNP detectors.

overall implementation complexity of the MA-PDNP detector is comparable to that of the corresponding PDNP detector.

The branch metric calculation requires the values of the data-dependent mean, variance and filter coefficients. These parameters are derived from the channel noise statistics which can be determined by means of a training sequence, or adaptively by using an algorithm such as that proposed in [7], [11], or a combination of these two methods. Of course, an adaptive scheme requires extra computation to determine the noise variances and filter coefficients using (9) and (10) at each time the noise statistics get updated.

## IV. SIMULATION RESULTS

For the NLTS calculation, we set the medium to soft-underlayer spacing to 20nm, and the medium thickness is set to 10nm. The channel spacing is 16nm, corresponding to a linear density of about  $1.59 \times 10^6$  bits/inch. The remanent magnetization to head field gradient ratio is set to 1.5. With these parameters, the NLTS of the isolated dibit pattern is about 20% of the channel bit spacing  $B$ .

The simulation uses pseudorandom input data divided into 5000-bit sectors. The equalizer utilizes the minimum mean-squared error (MMSE) monic constraint design [12]. The equalization target has length 3 and the number of FIR equalizer taps is set to 15. The noise statistics are obtained by means of a training sequence.

A comparison between the Viterbi detector, the PDNP detector, and the MA-PDNP detector is shown in Fig. 2. The normalized jitter noise variance is  $\sigma_J/B = 0.1$ . The bit density is set to  $T_{50}/B = 1$ . The BER for different detectors is plotted versus  $SNR_W$ . The PDNP detector and the MA-PDNP detector have the same number of states as the Viterbi detector,  $M = J = 2$ . The data-dependence parameter is set to  $L_x = 6$  for both the PDNP detector and the MA-PDNP detector. The two detectors also have the same noise prediction filter length  $L_y = 3$ . From the figure, it is clear that the MA-PDNP detector is superior to the PDNP detector, which, in turn, outperforms the conventional Viterbi detector. The intuitive explanation for this relative behavior is that the pattern-dependent noise prediction reduces the effect of the correlated jitter noise, while the pattern-dependent mean compensates for the NLTS.

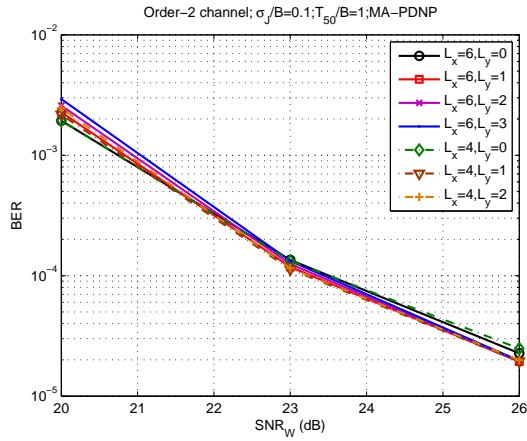


Fig. 3. Comparison between different  $L_y$  and  $L_x$ .

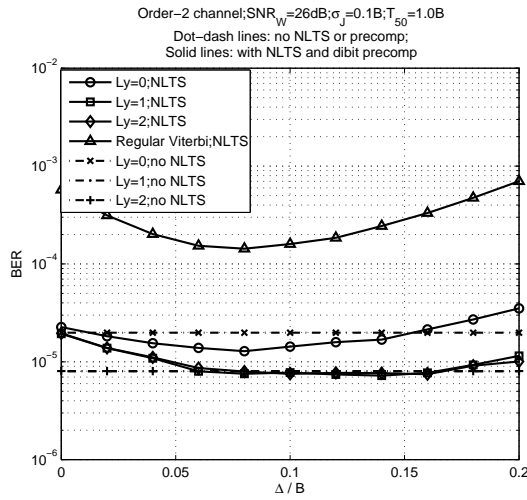


Fig. 4. MA-PDNP detector with dibit precompensation.

Fig. 3 compares MA-PDNP detector performance with various values of the noise prediction length  $L_y$  and the data-dependence length  $L_x$ , assuming  $M = J = 2$ . Note that the performance achieved with  $L_x = 4$  is very close to that obtained with  $L_x = 6$ . We also note that the BER at  $SNR_W = 26\text{dB}$  is nearly identical for  $L_y = 1, 2, 3$ . Even when we set  $L_y = 0$ , corresponding to no noise prediction, the BER is only slightly higher.

The MA-PDNP detector can also be used in combination with write precompensation. To illustrate this, we simulated the BER for the MA-PDNP detector used in conjunction with a dibit precompensation scheme, with the results shown in Fig. 4. The channel parameters were set to  $SNR_W = 26\text{dB}$ ,  $\sigma_J/B = 0.1$  and  $T_{50}/B = 1$ . The parameter settings for the NLTS calculation were the same as in the previously discussed simulations. We set  $L_x = 6$ , and used the same number of trellis states in the MA-PDNP detector as in the conventional Viterbi detector, i.e.,  $M = J = 2$ . The figure shows that for a channel with NLTS, the MA-PDNP detector can achieve a much lower BER than a Viterbi detector even with dibit precompensation. For prediction filter lengths  $L_y = 1, 2$ , we see that the MA-PDNP detector with dibit precompensation

achieves the same performance as that of a PDNP detector applied to a channel with no NLTS. This is true over a wide range of precompensation values.

## V. CONCLUSIONS

In this paper, we presented a mean-adjusted pattern-dependent noise-prediction (MA-PDNP) detector for perpendicular recording channels with nonlinear transition shift (NLTS), transition jitter, and additive Gaussian noise. The new detector reduces the performance degradation caused by data-dependent NLTS and media noise. According to simulation results for an order-2 channel approximation, the MA-PDNP detector improves the performance significantly as compared to both the conventional Viterbi detector and the PDNP detector. At the same time, the computational complexity of the MA-PDNP detector is comparable to that of the PDNP detector. The MA-PDNP detector can also be combined with write precompensation to achieve further performance improvements. Simulation results show that, when used with a simple dibit precompensation technique, the MA-PDNP detector provides the same BER performance as a PDNP detector applied to a channel with no NLTS, over a wide range of precompensation values.

## ACKNOWLEDGEMENT

The authors would like to thank the Information Storage Industry Consortium (INSIC) Extremely High Density Recording (EHDR) program and the Center for Magnetic Recording Research at UC San Diego for funding this work.

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