Information-Theoretic Limits of Two-Dimensional Optical Recording Channels

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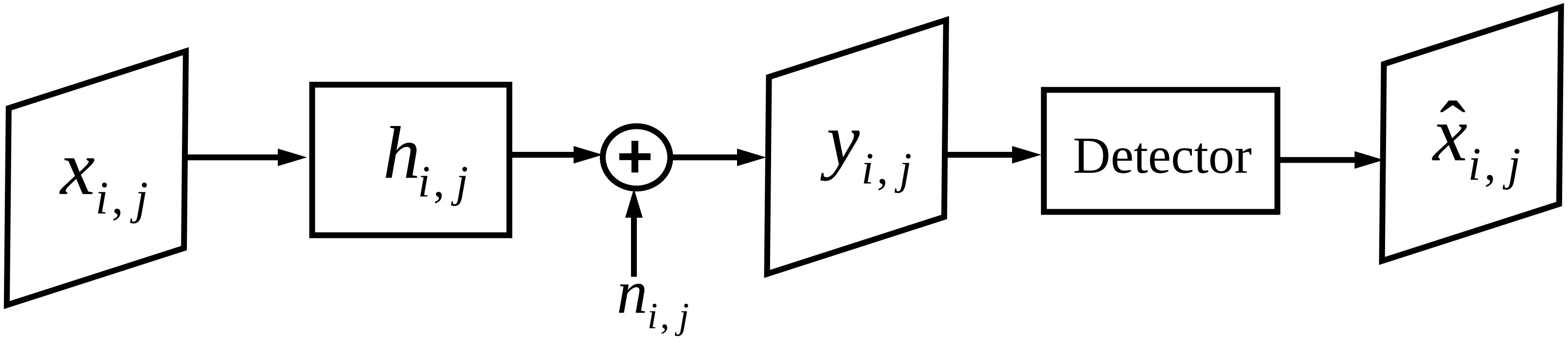
Outline

- Optical recording channel model
- Information rates and channel capacity
- Combined coding and detection
- Approaching information-theoretic limits
- Concluding remarks

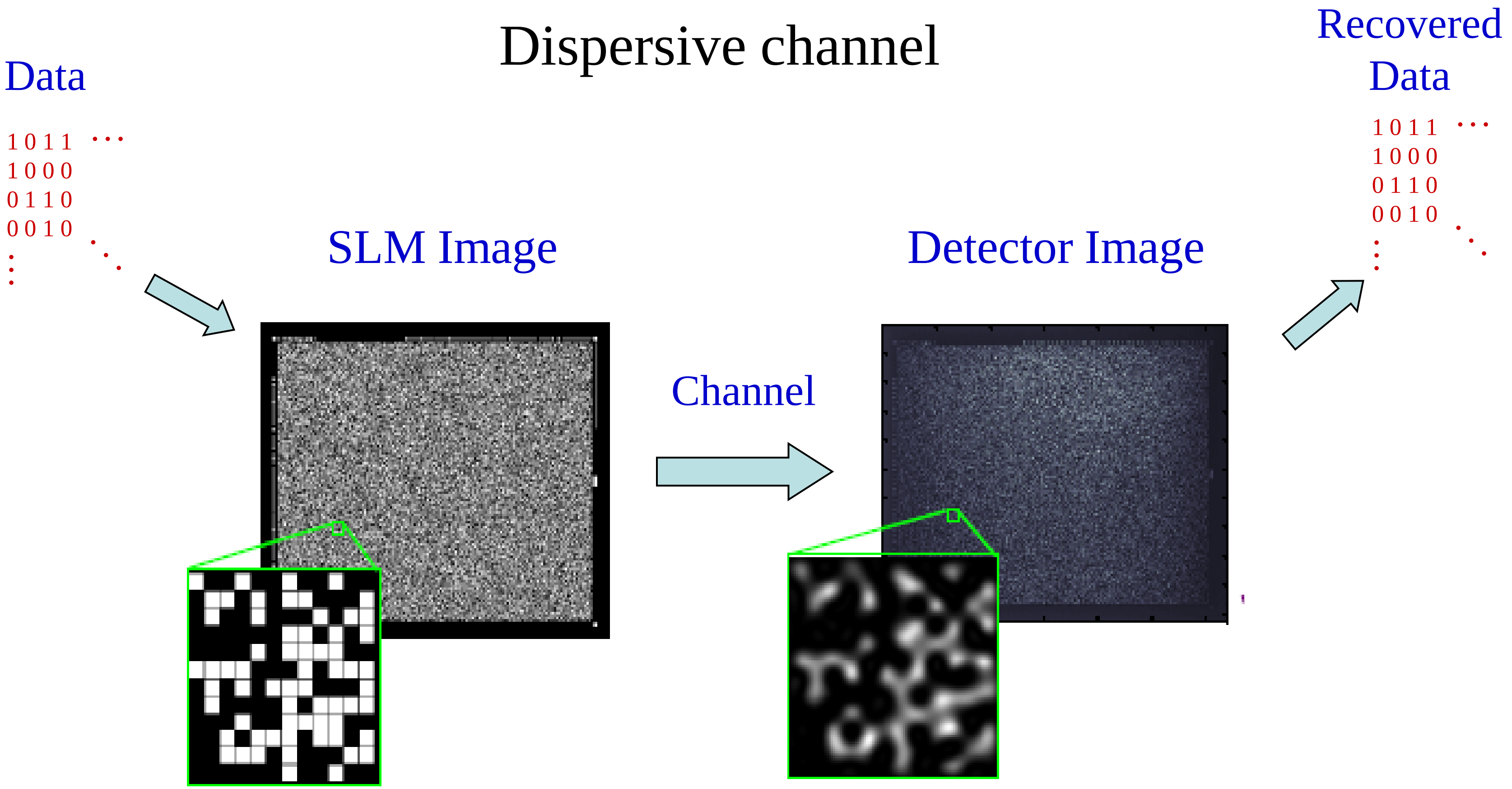
2D Optical Recording Model



- Binary data: $x_{i,j}$
- Linear intersymbol interference (ISI): $h_{i,j}$
- Additive white Gaussian noise: $n_{i,j}$
- Output:

$$y_{i,j} = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} h_{k,l} x_{i-k,j-l} + n_{i,j}$$

Holographic Recording



Courtesy of Kevin Curtis, InPhase Technologies

Holographic Channel

Recorded Impulse

0	0	0
0	**1**	0
0	0	0

Readback Samples

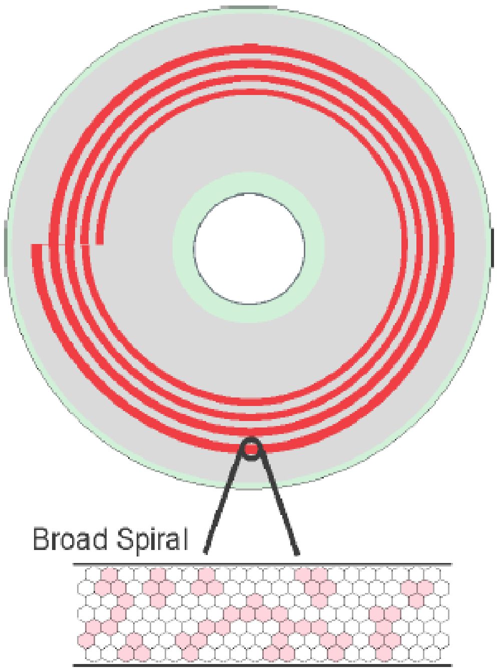
0	0	0
0	**1**	**1**
0	**1**	**1**

Normalized impulse response:

$$h_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

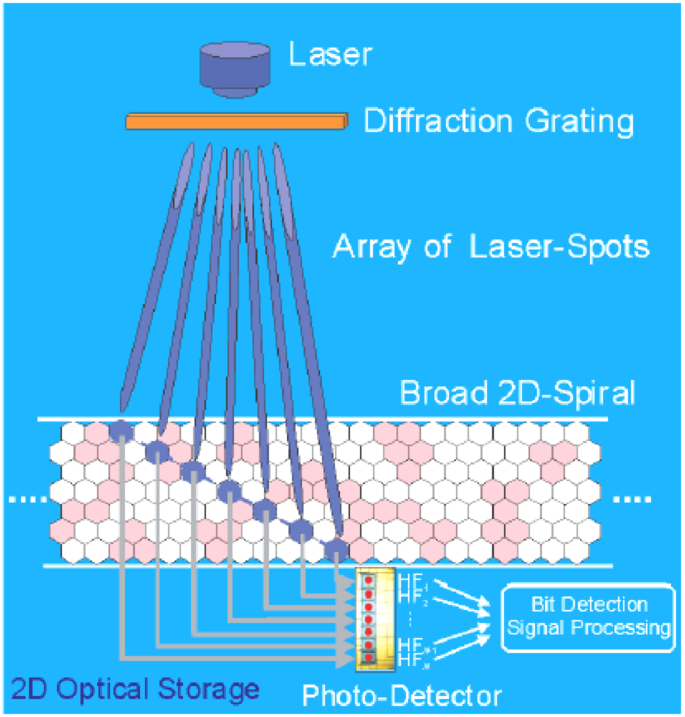
TwoDOS Recording

Disc



(spiral contains 11 bit-rows)

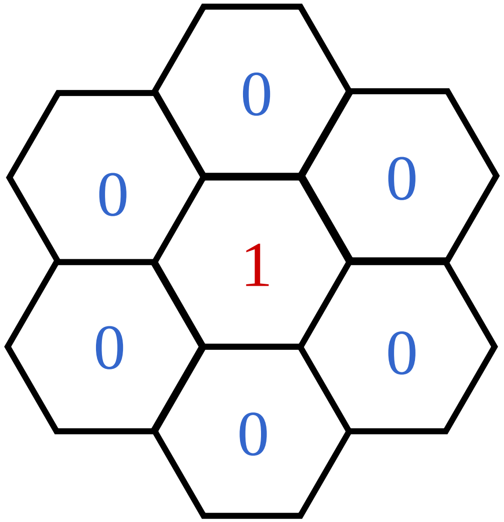
Readout



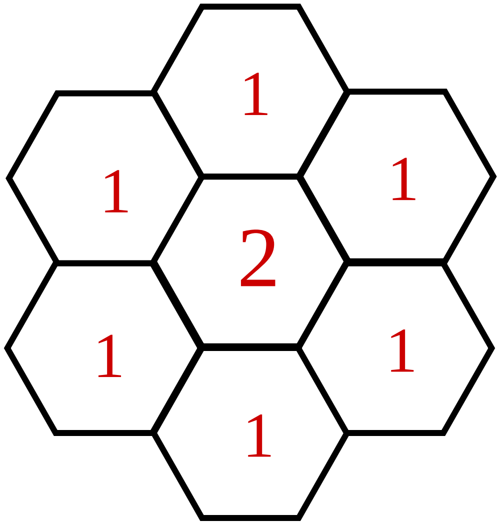
Courtesy of Wim Coene, Philips Research

TwoDOS Channel

Recorded Impulse



Readback Samples



Normalized impulse response:

$$h_2 = \frac{1}{\sqrt{10}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Channel Information Rates

- Capacity (C)
  - "The maximum achievable rate at which reliable data storage and retrieval is possible"
- Symmetric Information Rate (SIR)
  - "The maximum achievable rate at which reliable data storage and retrieval is possible using a linear code."

Objectives

- Given a binary 2D ISI channel:

1. Compute the SIR (and capacity) .

2. Find practical coding and detection algorithms that approach the SIR (and capacity) .

Computing Information Rates

- Mutual information rate:

$$I(X;Y) = H(Y) - H(Y \mid X) = H(Y) - H(N)$$

- Capacity:

$$C = \max_{P(X)} I(X;Y)$$

- Symmetric information rate (SIR):

$$SIR = I(X;Y)$$

where X is i.i.d. and equiprobable

Detour: One-dimensional (1D) ISI Channels

- Binary input process $x[i]$
- Linear intersymbol interference $h[i]$
- Additive, i.i.d. Gaussian noise $n[i] \sim N(0, \sigma^2)$

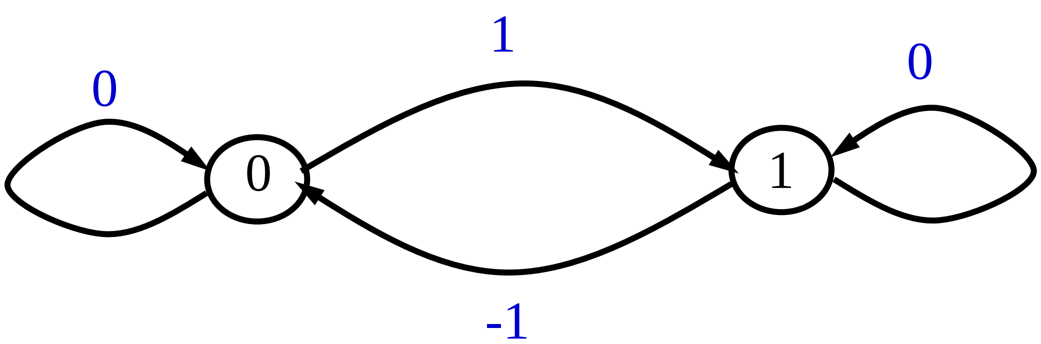
$$y[i] = \sum_{k=0}^{n-1} h[k]\, x[i-k] + n[i]$$

Example: Partial-Response Channels

- Common family of impulse responses:

$$h(D) = \sum_{i=0}^{N} h[i]\, D^{i} = (1-D)(1+D)^{N-1}$$

- Dicode channel $\quad h(D) = (1-D) \qquad h_{dicode} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \end{bmatrix}$



Entropy Rates

- Output entropy rate: $$H(Y) = \lim_{n\to\infty} \frac{1}{n} H\left(Y_1^n\right)$$
- Noise entropy rate: $$H(N) = \frac{1}{2} log(\pi e N_0)$$
- Conditional entropy rate:

$$H(Y \mid X) = \lim_{n\to\infty} \frac{1}{n} H\left(Y_1^n \mid X_1^n\right) = H(N)$$

Computing Entropy Rates

- Shannon-McMillan-Breimann theorem implies

$$-\frac{1}{n} log\ p\left(y_1^n\right) \underset{a.s.}{\longrightarrow} H\left(Y\right)$$

as $n \to \infty$, where y_1^n is a single long sample realization of the channel output process.

Computing Sample Entropy Rate

- The forward recursion of the sum-product (BCJR) algorithm can be used to calculate the probability $p(y_1^n)$ of a sample realization of the channel output.

- In fact, we can write

$$-\frac{1}{n} \log p\left(y_1^n\right) = -\frac{1}{n} \sum_{i=1}^{n} \log p\left(y_i \mid y_1^{i-1}\right)$$

where the quantity $p\left(y_i \mid y_1^{i-1}\right)$ is precisely the normalization constant in the (normalized) forward recursion.

Computing Information Rates

- Mutual information rate:

$$I(X;Y) = \boxed{H(Y)} - \boxed{H(N)}$$

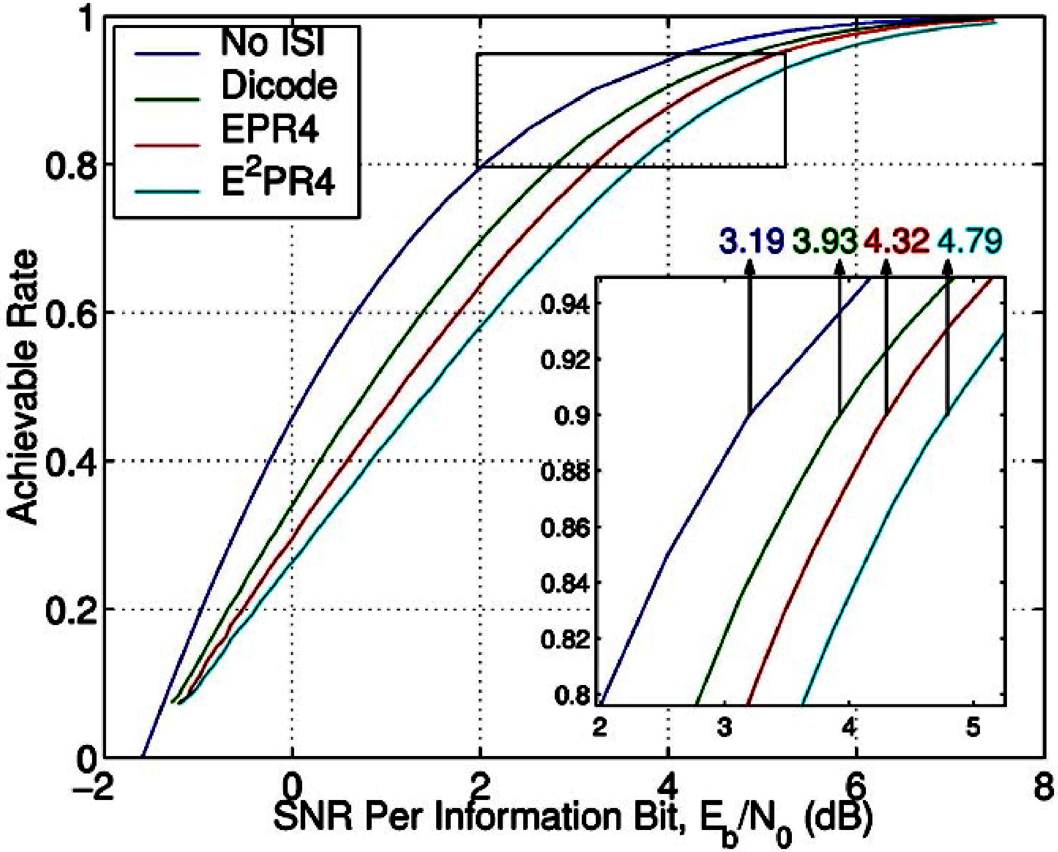
computable for given X

known

- $SIR = I(X;Y)$ where X is i.i.d. and equiprobable

- Capacity: $C = \max_{P(X)} I(X;Y)$

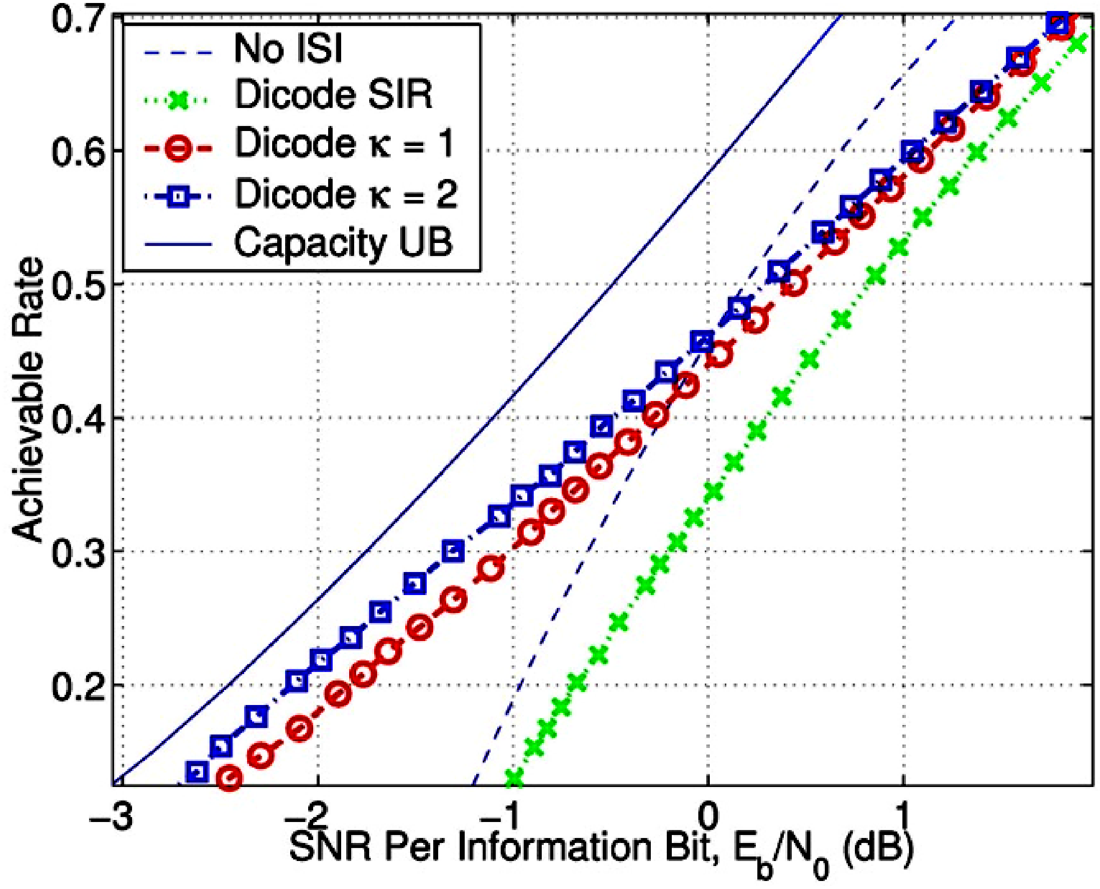
SIR for Partial-Response Channels



Computing the Capacity

- For Markov input process of specified order *r* , this technique can be used to find the mutual information rate. (Apply it to the combined source-channel.)
- For a fixed order *r* , [Kavicic, 2001] proposed a Generalized Blahut-Arimoto algorithm to optimize the parameters of the Markov input source.
- The stationary points of the algorithm have been shown to correspond to critical points of the information rate curve [Vontobel,2002] .

Capacity Bounds for Dicode h(D)=1-D



Approaching Capacity: 1D Case

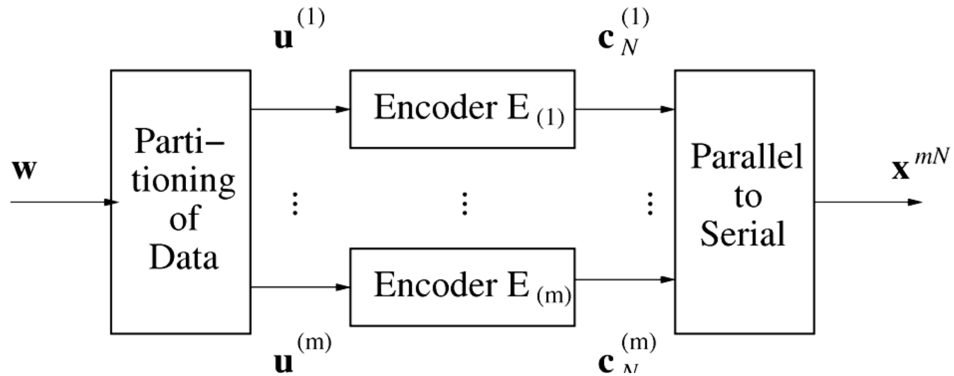
- The BCJR algorithm, a trellis-based “forward-backward” recursion, is a practical way to implement the optimal *a posteriori* probability (APP) detector for 1D ISI channels.

- Low-density parity-check (LDPC) codes in a multilevel coding / multistage decoding architecture using the BCJR detector can operate near the SIR.

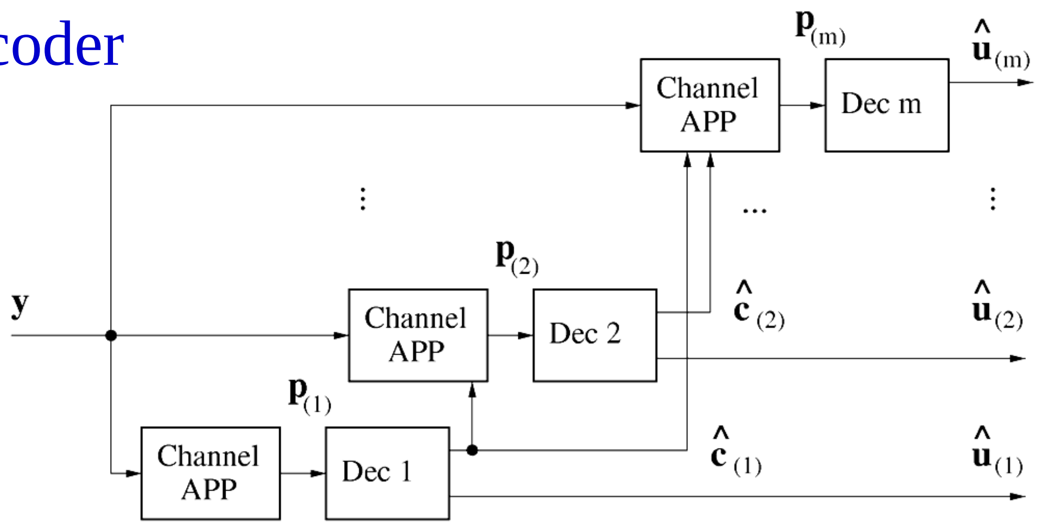
Multistage Decoder Architecture

Multilevel encoder

$$R_{av,m} = \frac{1}{m}\sum_{i=1}^{m} R_m^{(i)}$$



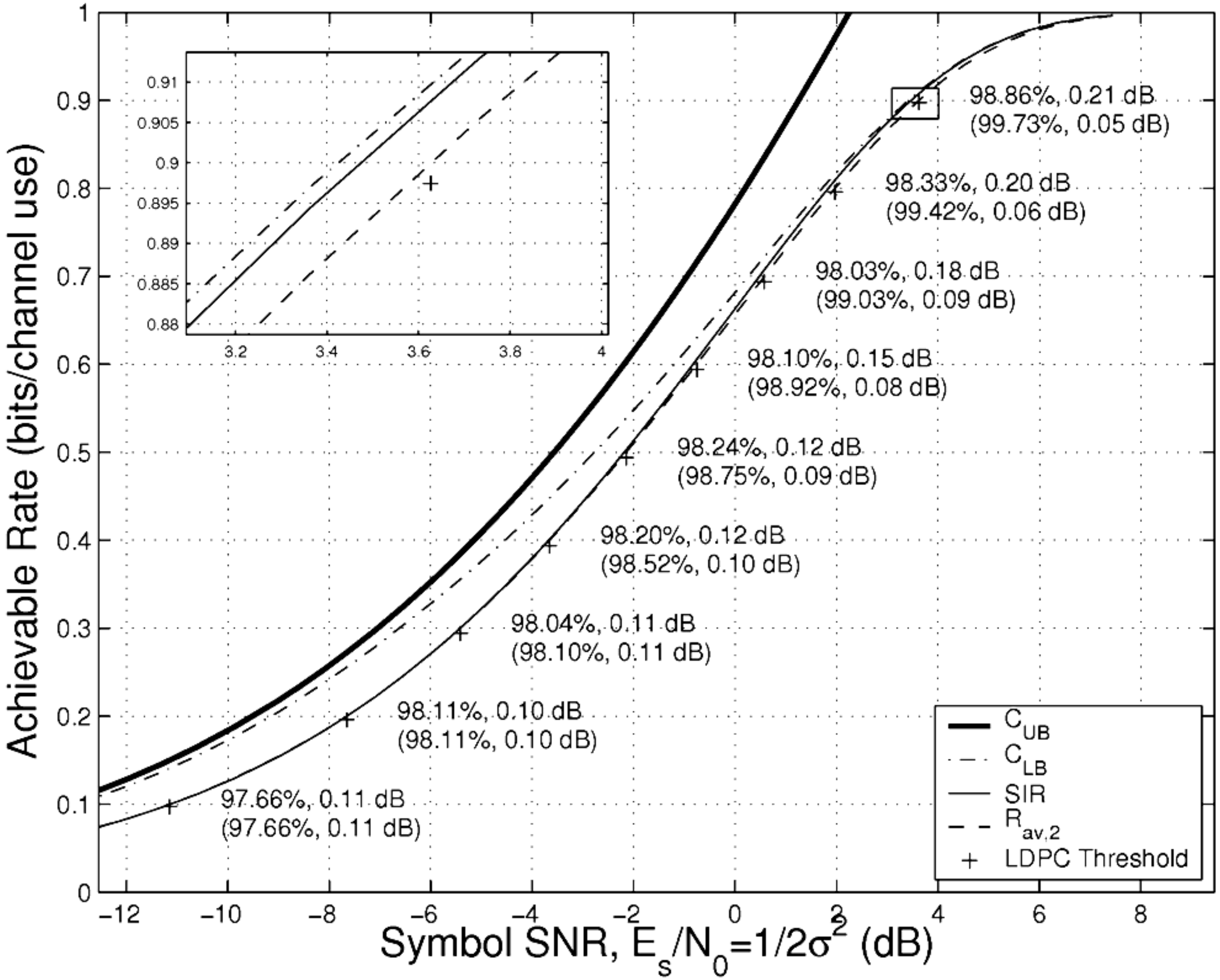
Multistage decoder



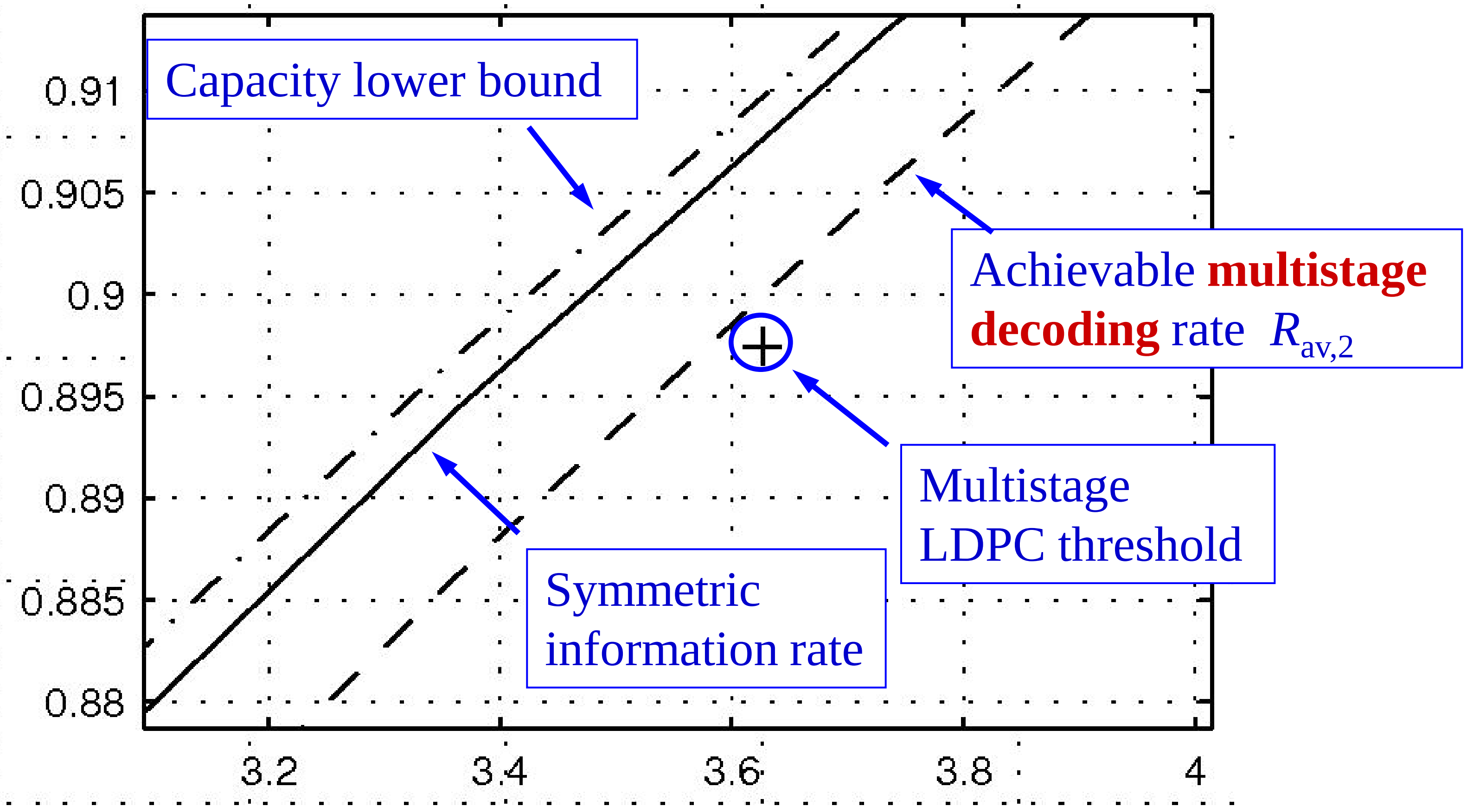
Multistage Decoding (MSD)

- The maximum achievable sum rate $R_{\mathrm{av},m} = \frac{1}{m}\sum_{i=1}^{m} R_m^{(i)}$ with multilevel coding (MLC) and multistage decoding (MSD) approaches the SIR on 1D ISI channels, as $m \rightarrow \infty$.
- LDPC codes optimized using density evolution with design rates close to $R_m^{(i)},\ i = 1,\ldots,m$ yield thresholds near the SIR.
- For 1D channels of practical interest, m need not be very large to approach the SIR.

Information Rates for Dicode



Information Rates for Dicode



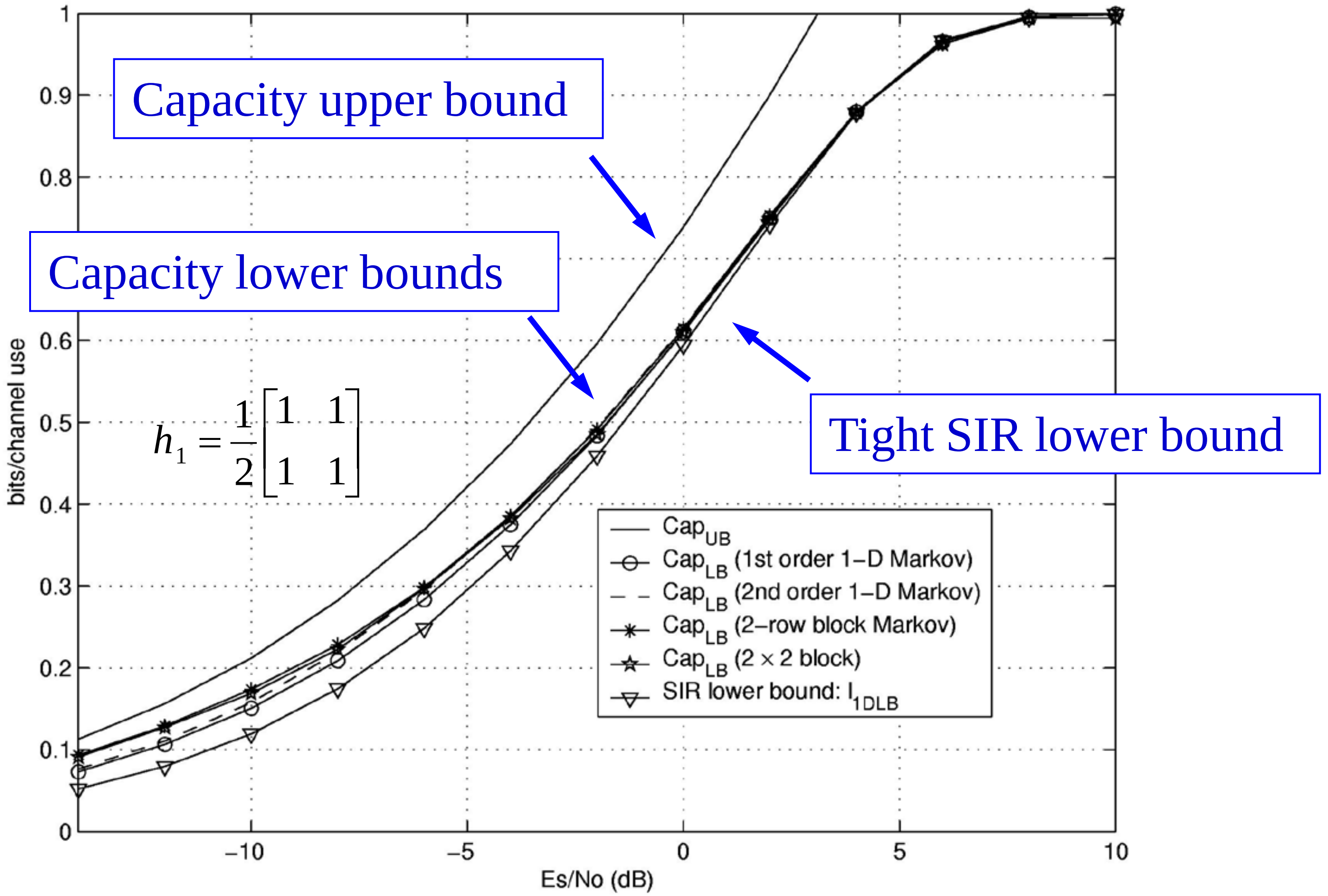
Back to the Future: 2D ISI Channels

- In contrast, in 2D, there is
  - no simple calculation of the $H(Y)$ from a large channel output array realization to use in information rate estimation.
  - no known analog of the BCJR algorithm for APP detection.
  - no proven method for optimizing an LDPC code for use in a detection scheme that achieves information-theoretic limits.
- Nevertheless…

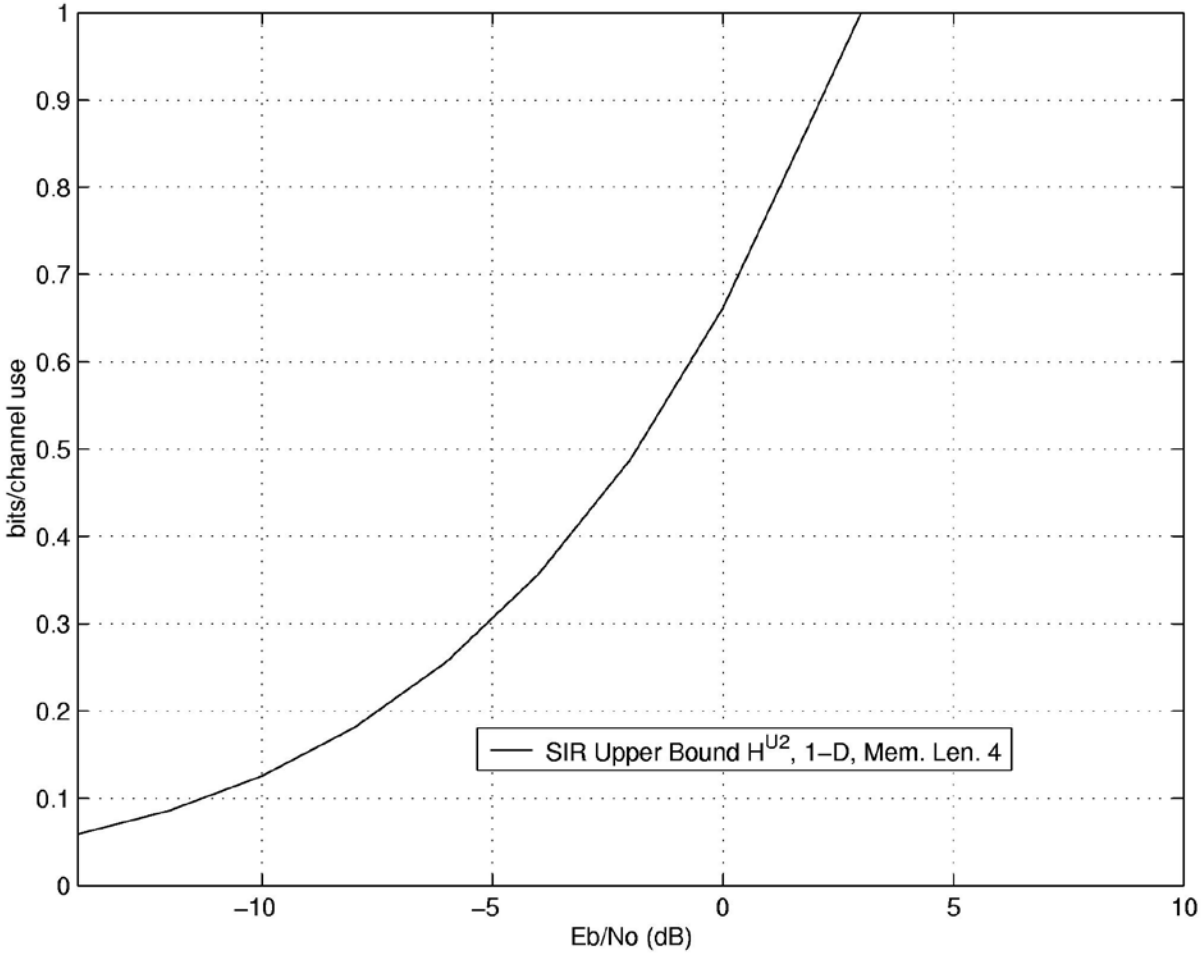
Bounds on the 2D SIR and Capacity

- Methods have been developed to bound and estimate, sometimes very closely, the SIR and capacity of 2D ISI channels, using:
  - Calculation of conditional entropy of small arrays
  - 1D "approximations" of 2D channels
  - Generalizations of certain 1D ISI bounds
  - Generalized belief propagation

Bounds on SIR and Capacity of h_1



Bounds on SIR of h_2



2D Detection – IMS Algorithm

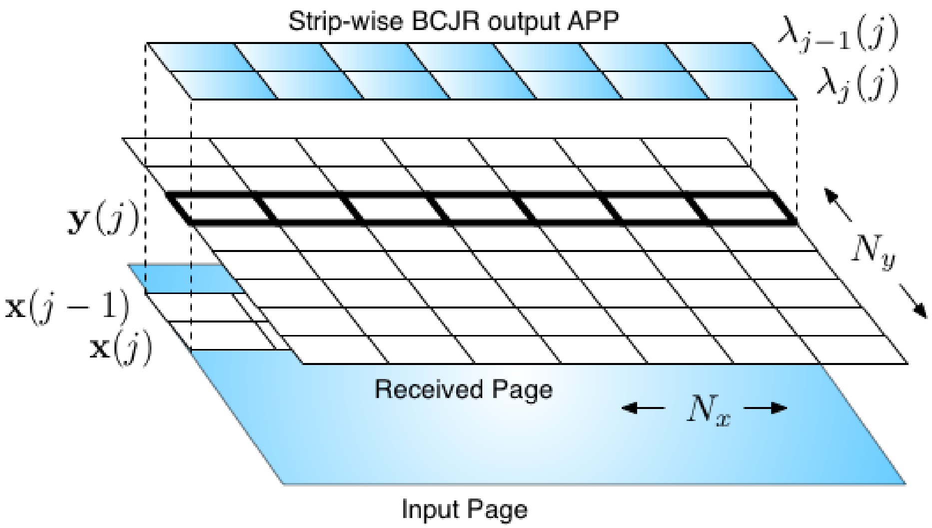
- Iterative multi-strip (IMS) detection offers near-optimal bit detection for some 2D ISI channels.
- Finite computational complexity per symbol.
- Makes use of 1D BCJR algorithm on "strips".
- Can be incorporated into 2D multilevel coding, multistage decoding architecture.

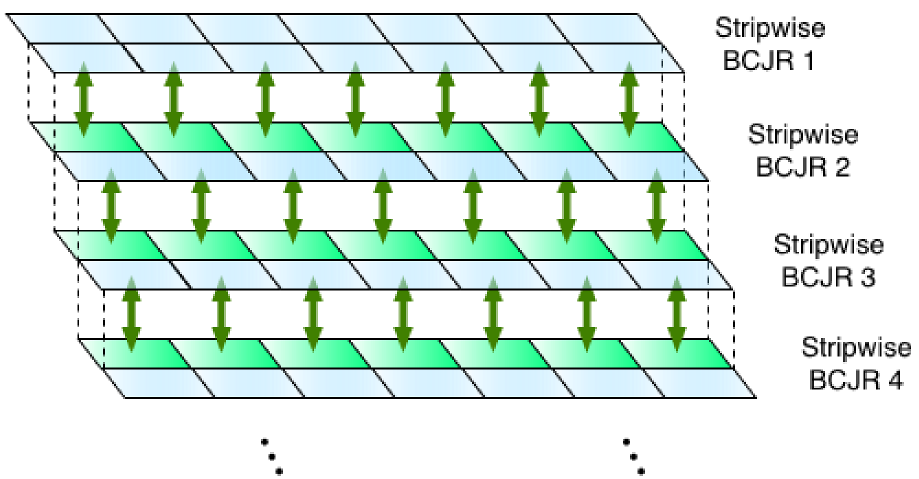
Iterative Multi-Strip (IMS) Algorithm

Step 1. Use 1D BCJR to decode strips.

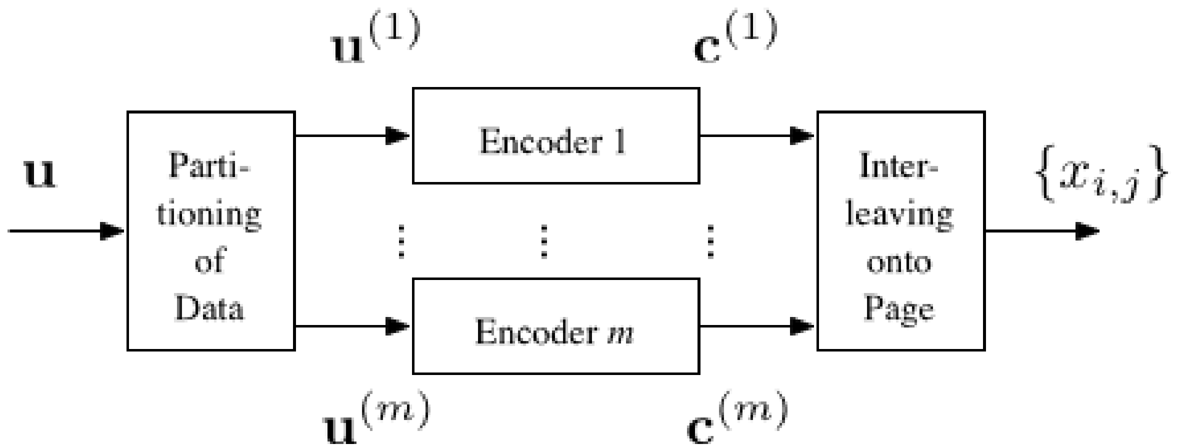
iterate

Step 2. Pass extrinsic information between *overlapping* strips.

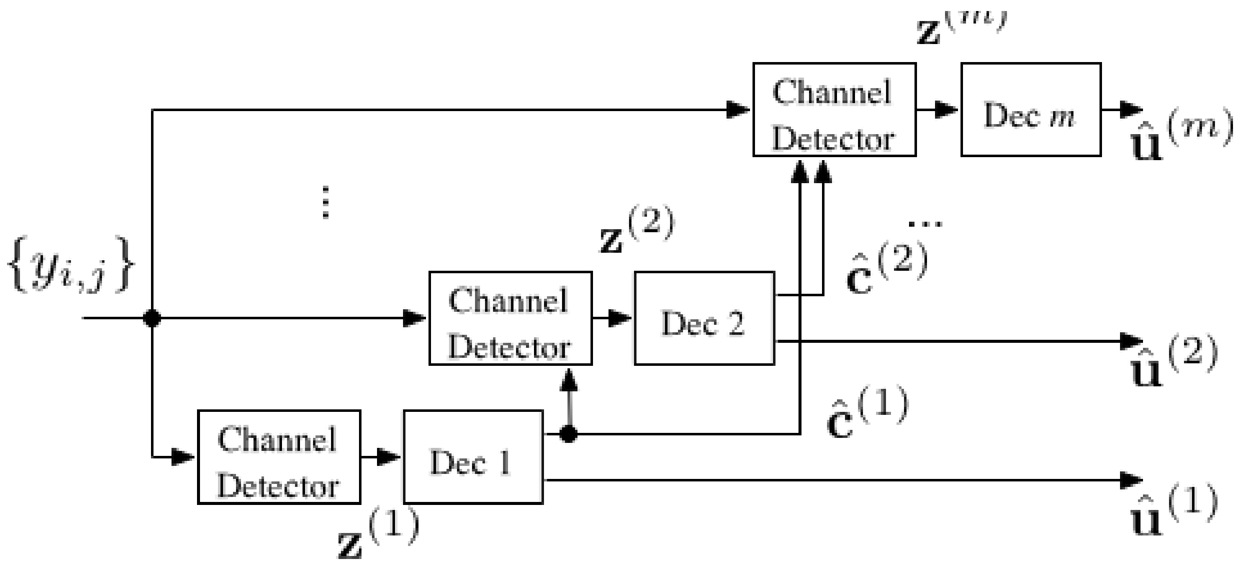




2D Multistage Decoding Architecture

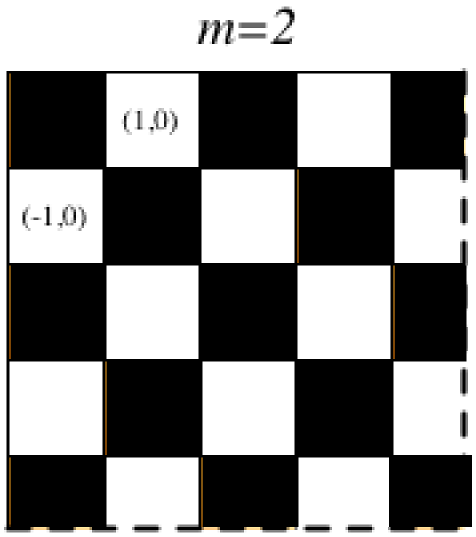


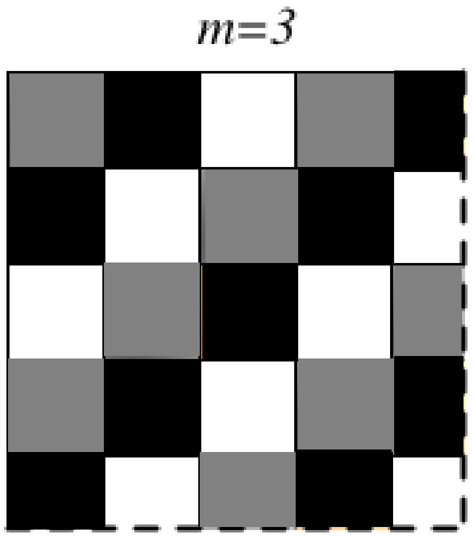
Previous stage
decisions pin trellis
for strip-wise
BCJR detectors



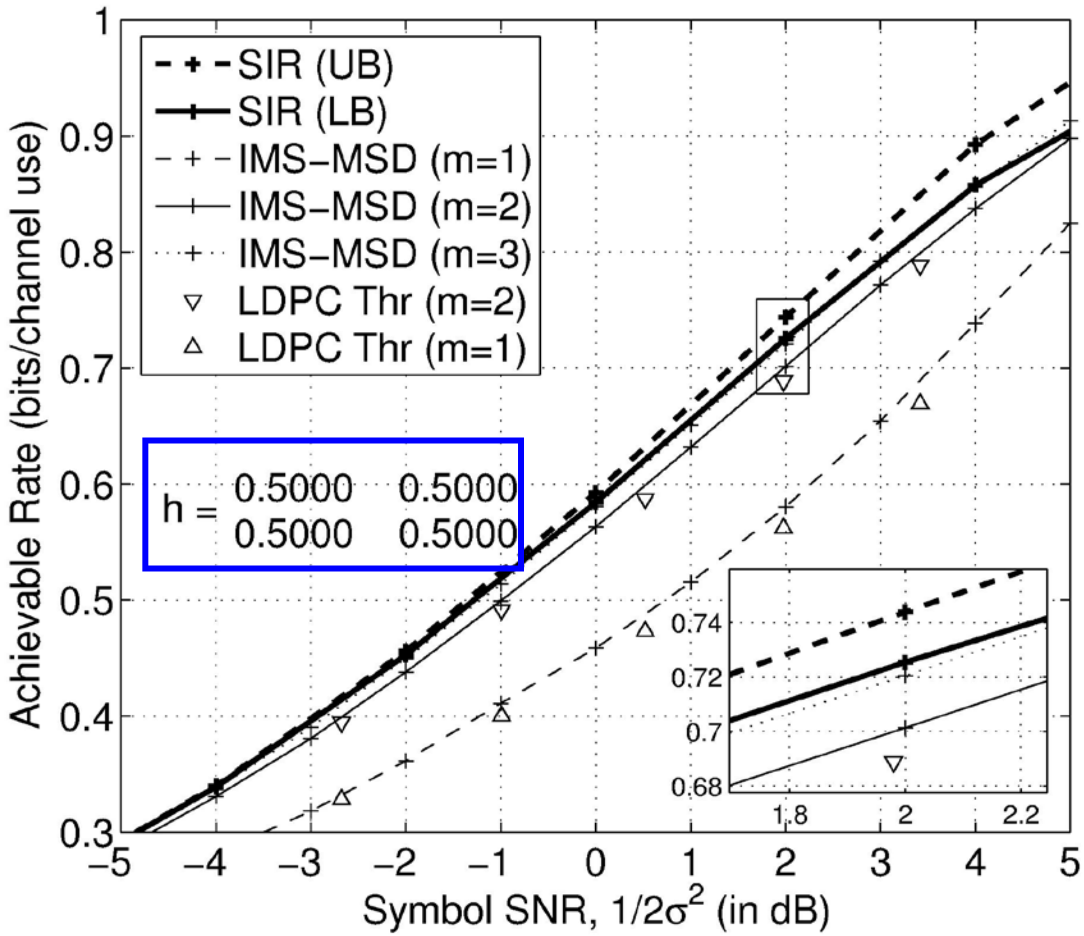
2D Interleaving

- Examples of 2D interleaving with $m=2,3$.

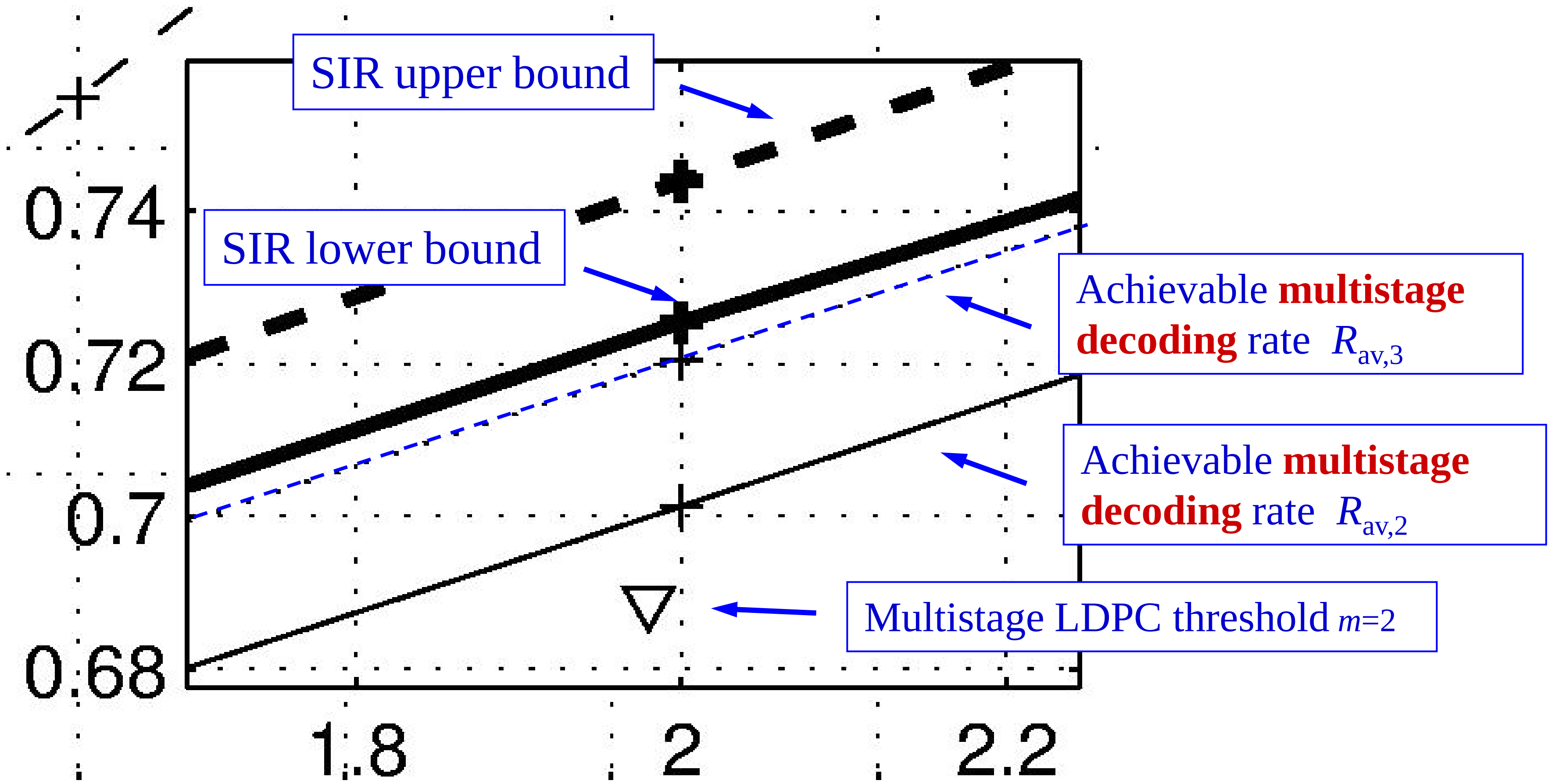




IMS-MSD for h_1



IMS-MSD for h_1



Alternative LDPC Coding Architectures

- LDPC (coset) codes can be optimized via "generalized density evolution" for use with a 1D ISI channel in a "turbo-equalization" scheme.

- LDPC code thresholds are close to the SIR.

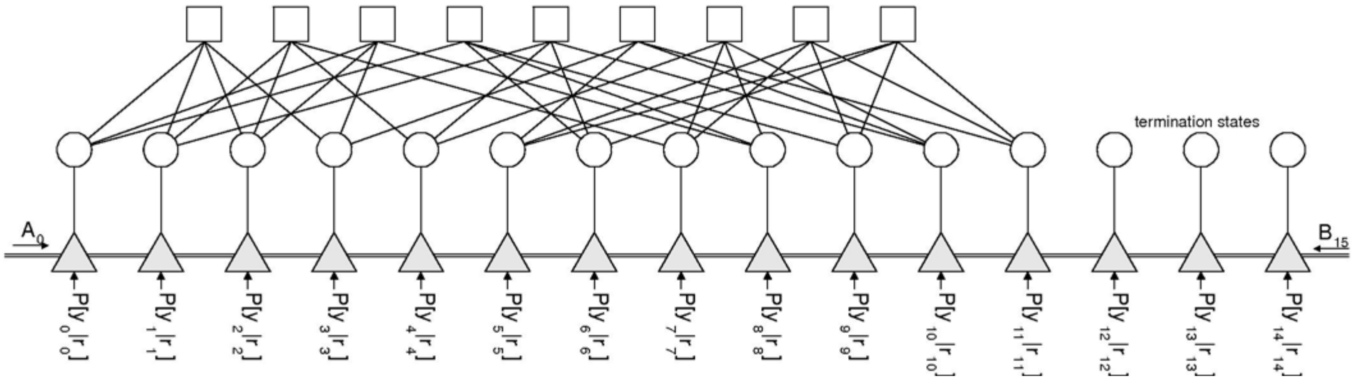
- This "turbo-equalization" architecture has been extended to 2D, but "2D generalized density evolution" has not been rigorously analyzed.

1D Joint Code-Channel Decoding Graph

check nodes

variable nodes

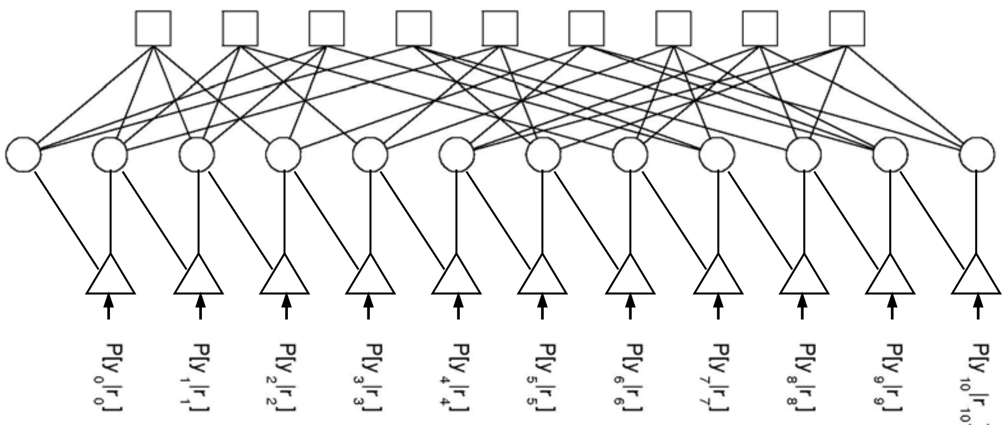
channel state nodes
(BCJR)



check nodes

variable nodes

channel output nodes
(MPPR)

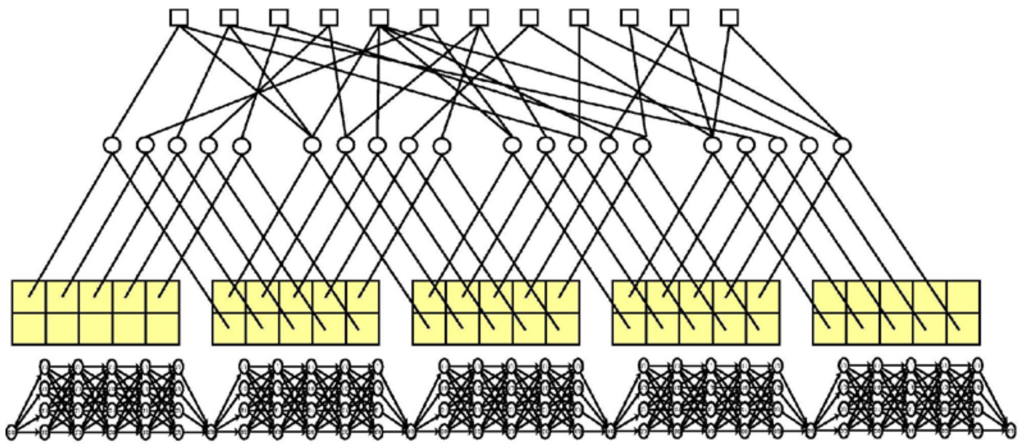


2D Joint Code-Channel Decoding Graph

check nodes

variable nodes

channel state nodes
(IMS)

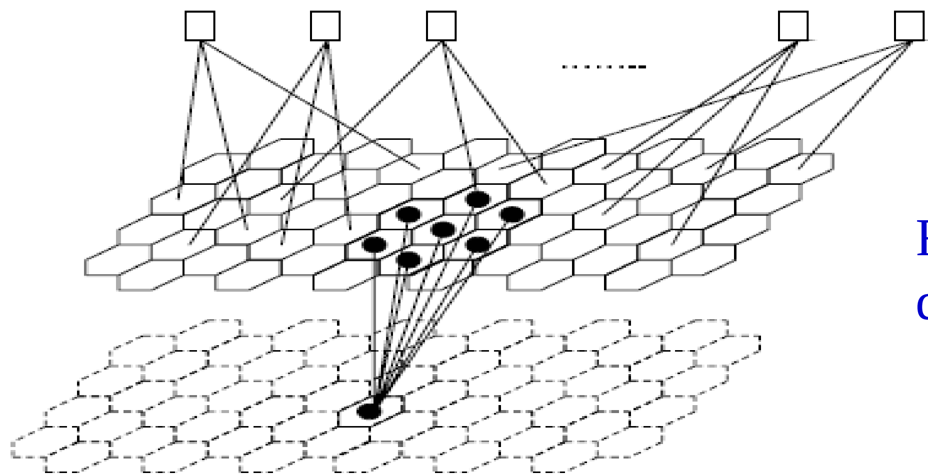


IMS
detector

check nodes

variable nodes

channel output nodes
(“Full graph”)



Full graph
detector

Concluding Remarks

- For 2D ISI channels, the following problems are hard:
  - Bounding and computing achievable information rates
  - Optimal detection with acceptable complexity
  - Designing codes and decoders to approach limiting rates
- But progress is being made, with possible implications for design of practical 2D optical storage systems.