Performance Analysis of Turbo-Equalized Dicode Partial-Response Channel

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Abstract

This paper addresses the performance of maximum-likelihood decoding of a serial concatenation comprising a high-rate convolutional code or a turbo code, a uniform interleaver, and a dicode partial-response channel. The effect of a channel precoder on the system performance is also considered. Bit- and word-error-rate estimates based upon properties of the average Euclidean distance spectrum of the coded partial-response channel are derived. The estimates are compared to computer simulation results, and implications for system design are discussed.

1 Introduction

Trellis coding techniques that improve the reliability of binary input-constrained, intersymbol interference (ISI) channels are of interest in both digital communications and data storage applications. Drawing inspiration from the success of turbo codes [1], [2], several authors have recently considered iterative decoding architectures for coding schemes of the form depicted in Fig. 1, where the outer encoder is a convolutional or turbo encoder, π is an interleaver, $g(D)$ represents a precoder function, and $h(D)$ is the channel transfer polynomial.

This system resembles serial concatenation of interleaved codes, investigated by Benedetto, *et al.* [3], with the inner code replaced by the ISI channel. For such a system, Douillard, *et al.* [4] presented an iterative receiver structure, dubbed "turbo-equalization," to combat ISI due to multipath effects on convolutionally-coded Gaussian and Rayleigh transmission channels. They introduced an interleaver between the encoder and channel, and, as in turbo decoding, soft-output decisions from the channel detector and from the convolutional decoder are used in an iterative and cooperative fashion.

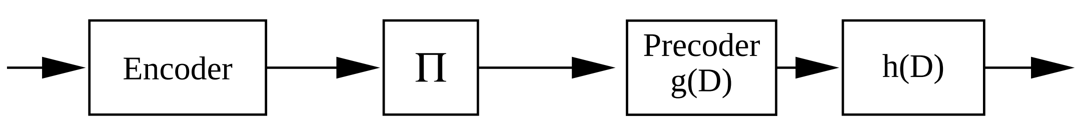

Figure 1: Trellis coded partial response system.

Motivated largely by the potential applications to digital magnetic recording, several authors have explored the application of turbo coding methods to the dicode and class-4 partial-response (PR4) channels, which have transfer functions $h(D) = 1 - D$ and $h(D) = 1 - D^2$, respectively. Heegard [5] and Pusch, *et al.* [6] illustrated the design and iterative decoding of turbo codes for the dicode channel, using rates 1/2 and lower. Reed and Schlegel [7], extending prior results on a low-complexity, iterative multiuser receiver structure with interference cancellation, have evaluated the benefits of turbo-equalization for a rate 1/2, convolutionally-coded, PR4 channel and E^2PR4 channel.

Ryan, *et al.* [8], and Ryan [9], demonstrated that by using as an outer code a parallel-concatenated turbo code, punctured to achieve rates 4/5, 8/9 and 16/17 typical of commercial magnetic recording systems, one could obtain significant coding gain relative to previously known high-rate trellis-coding techniques on a precoded dicode or PR4 channel.

Recently, Souvignier, *et al.* [10] considered serial concatenated systems similar to that addressed in [8],[9]. On a precoded dicode channel, the performance achievable with a high-rate outer convolutional code, rather than a turbo code, was investigated by means of computer simulation. Somewhat surprisingly, the convolutional code was found to perform as well as the turbo code. Moreover, removal of the channel precoder was found to improve the performance of the turbo-coded system at low SNR, while degrading the performance of the convolutionally-coded system.

This paper was motivated, in part, by the desire to understand better the empirically observed differences in error-rate in the precoded and non-precoded serial concatenated systems. We address the performance of maximum-likelihood decoding of a serial concatenated system as shown in Fig. 1, comprising a high-rate convolutional encoder or a parallel concatenation of two convolutional encoders, an interleaver, and a dicode partial-response channel, with and without a channel precoder.

The maximum–likelihood (ML) union bound on word error rate (WER) for a block-coded, additive white Gaussian noise (AWGN) channel can be expressed as [11]

$$P_w \leq \sum_{d_E=d_{min}}^{\infty} \overline{T}(d_E) Q\left(\frac{d_E}{2\sigma}\right), \tag{1}$$

where d_E denotes Euclidean distance between two channel output words, $\overline{T}(d_E)$ denotes the average Euclidean weight enumerator which is the average number of codewords with Euclidean distance d_E from the output of a given codeword, and σ^2 denotes the noise variance on the channel. The corresponding bit error rate (BER) bound is [11]

$$P_b \leq \sum_{d_E=d_{min}}^{\infty} \frac{\overline{T}(d_E)\overline{w}(d_E)}{K} Q\left(\frac{d_E}{2\sigma}\right), \tag{2}$$

where $\overline{w}(d_E)$ denotes the average information Hamming distance to codewords whose outputs have distance d_E, and K denotes the number of information bits in a codeword.

For an exact analysis, the full compound error-event characterization for a code interleaved and concatenated with the ISI channel must be determined. The complexity of this computation is generally prohibitively high. To overcome this difficulty, we introduce a technique for computing an approximation to the average weight enumerator $\overline{T}(d_E)$ for a high-rate, coded partial response channel. The result depends only upon the output Hamming weight enumerator of the outer code,

$$\overline{A}(d) = \sum_{i=0}^{K} \overline{A}(d, i), \tag{3}$$

where $\overline{A}(d,i)$ denotes the average number of error words of Hamming output weight d and input weight i.

In Section 2, we present the analysis for the dicode channel, first without a precoder, then with a precoder of the form $g(D) = 1/(1 \oplus D)$. In Section 3, we consider systems incorporating a rate 8/9 outer punctured convolutional code and a rate 4/5 turbo code. The analytical performance estimates are compared to the results of computer simulation. Section 4 concludes the paper.

2 Error Event Analysis on the Dicode Channel

Referring to the system model in Fig. 1, we assume that the encoder is a block encoder, for example, a truncated convolutional encoder or a turbo encoder. Let $\mathbf{b} = [b_1, \ldots, b_N]$ denote a codeword, and $\mathbf{c} = \pi(\mathbf{b})$ be the corresponding output of the interleaver. The output of the precoded channel is denoted $\mathbf{x} = [x_1, \ldots, x_N]$.

Given two codewords $\mathbf{b_1}$ and $\mathbf{b_2}$, let $\mathbf{e} = \mathbf{b_1} \oplus \mathbf{b_2}$ be the corresponding Hamming error word, and similarly define the interleaved Hamming error word $\mathbf{f} = \mathbf{c_1} \oplus \mathbf{c_2}$. The Euclidean error word is given by $\zeta = \mathbf{c_1} - \mathbf{c_2}$ and the corresponding output error word is $\chi = \mathbf{x_1} - \mathbf{x_2}$.

We will make two simplifying assumptions in the analysis of the system performance. First, we assume that the interleaver π is a uniform interleaver, as defined by Benedetto et al. [12]. Second, we make the assumption that, for any codeword $\mathbf{e}$, the contribution to $\overline{T}(d_E)$ of all error words $\epsilon = \mathbf{b_1} - \mathbf{b_2}$, where $\mathbf{b_1} = \mathbf{b_2} \oplus \mathbf{e}$, is approximately equal to the contribution of the set of error words produced when $\mathbf{b_1}$ and $\mathbf{b_2}$ are not restricted to lie within the code. This is equivalent to treating the permuted code bits within an error event at the output of the interleaver as independent and identically distributed (i.i.d.), with equiprobable bit values. The resulting estimate of the contribution to the Euclidean weight enumerator therefore depends only upon the Hamming weight of the word $\mathbf{e}$. The rationale behind this second assumption is that the system uses a very high-rate linear encoder, in tandem with the uniform interleaver.

In Section 2.1, we investigate the relationship between the Hamming weight $d_H(\mathbf{f})$ of the interleaved Hamming error word and the squared Euclidean distance $d_E^2(\zeta) = \sum_{i=1}^{N} {\chi_i}^2$ of the corresponding output error word on the dicode channel, $h(D) = 1 - D$. We then examine the distribution of error events induced by the action of the uniform interleaver. Using these results, we then derive an estimate for the system performance.

In Section 2.2, we derive the corresponding result for the dicode channel with a precoder characterized by $g(D) = 1/(1 \oplus D)$.

2.1 Dicode Channel With No Precoder

2.1.1 Error Event Distance Properties

Fig. 2 shows a trellis section for the dicode channel with no precoder. The branch labels are of the form c_i/x_i, where c_i is the input to the channel at time i, and x_i is the corresponding channel output.

Let f be an error word with Hamming weight $l = d_H(f)$, corresponding to a possibly compound input error event. Referring to Fig. 2, and assuming a fixed initial state for all codewords, f can be uniquely decomposed into a concatenation of disjoint error sub-events f_i, $i = 1, \ldots, m$, for some $m \geq 1$, consisting of one or more consecutive errors.

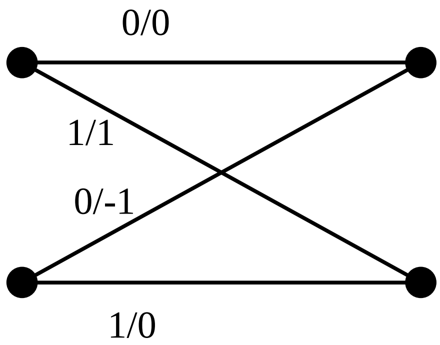

Figure 2: Trellis section for the dicode channel.

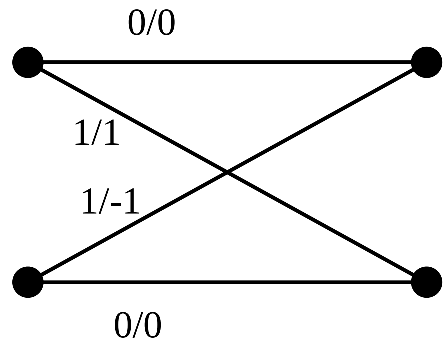

Figure 3: Trellis section for the precoded dicode channel.

Letting $l_i = d_H(f_i)$ denote the number of errors in the sub-event, we have $l = \sum_{i=1}^{m} l_i$. For $i \le m-1$, the sub-event f_i corresponds to a simple closed error event on the trellis, diverging from and remerging with the correct path, with no common intermediate states. For $i = m$, the corresponding sub-event f_m may be either closed or open; in the latter case the paths diverge and never remerge.

Let j_i denote the bit position at which the sub-event f_i begins. For a closed sub-event, the contribution to the squared Euclidean distance is given by

$$d_E^2(f_i) = 2 + 4 \sum_{k=j_i+1}^{j_i+l_i-1} c_k \oplus c_{k-1}. \tag{4}$$

If f_m is open, the contribution is

$$d_E^2(f_m) = 1 + 4 \sum_{k=j_m+1}^{j_m+l_m-1} c_k \oplus c_{k-1}. \tag{5}$$

The compound error event f generates squared Euclidean distance

$$d_E^2(f) = \sum_{i=1}^{m} d_E^2(f_i) = 2m + 4 \sum_{i=1}^{m} \sum_{k=j_i+1}^{j_m+l_m-1} c_k \oplus c_{k-1} - \delta(j_m + l_m - 1 - N), \tag{6}$$

where

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Invoking the assumption regarding the distribution of code bit values in the error events – namely, that their values are i.i.d. and equiprobable – we obtain an approximate contribution of an error word f of Hamming weight d to the average Euclidean distance spectrum. For example, for the case when f_m is closed, the result is:

$$Pr(d_E^2(f) = 2m + 4y | d, m, f_m \text{ closed}) = \binom{d-m}{y} 0.5^{d-m}. \tag{8}$$

The derivation of the approximate contribution for the case where f_m is open is similar.

2.1.2 Sub-Event Distribution at the Interleaver Output

Let e be an error word with Hamming weight $d_H(e) = d$. A specified interleaver will map e into an error word f which can be decomposed into m error sub-events, $f_i,\ i = 1, \ldots, m$, with corresponding weights l_i satisfying $l = \sum_{i=1}^{m} l_i$, as described above. A uniform interleaver maps the error word e into all $\binom{N}{d}$ possible error words f with equal probability $1/\binom{N}{d}$. In this section, we determine the distribution of the number m of sub-events of f, conditioned upon the Hamming weight d of the error word e, under the action of the uniform interleaver.

There are $\binom{d-1}{m-1}$ distinct decompositions of a sequence of d elements into m subsequences, each of length at least 1. The number of configurations in which these m subsequences can occur in a word of length N, with consecutive subsequences separated by at least one position, is given by $\binom{N-d+1}{m}$, so there are $\binom{N-d+1}{m}\binom{d-1}{m-1}$ weight d words with m sub-events. Taking into consideration the nature of the sub-event f_m, we can compute the conditional joint probabilities:

$$Pr(m, f_m \; closed|d) = \frac{\binom{N-d}{m}\binom{d-1}{m-1}}{\binom{N}{d}} \tag{9}$$

and

$$Pr(m, f_m \; open|d) = \frac{\binom{N-d}{m-1}\binom{d-1}{m-1}}{\binom{N}{d}}. \tag{10}$$

2.1.3 Approximation of the Euclidean Weight Enumerator

The approximation $\tau(d_E)$ to the average Euclidean weight enumerator $\overline{T}(d_E)$ can be computed by substituting (8) and its counterpart for the open case, along with (9) and (10) into

$$\begin{aligned}
\overline{T}(d_E) &= \sum_{k=1}^{N} \overline{A}(k) Pr\left(d_E|d_H = k\right) \\
&= \sum_{k=1}^{N} \overline{A}(k) \sum_{m=1}^{k} \left(Pr\left(d_E|d_H = k, m, f_m \; closed\right) Pr\left(m, f_m \; closed|d_H = k\right)\right. \\
&+ \left. Pr\left(d_E|d_H = k, m, f_m \; open\right) Pr\left(m, f_m \; open|d_H = k\right)\right).
\end{aligned} \tag{11}$$

The approximate average input error weight enumerator is similarly computed by substitution into

$$\overline{w}(d_E) = \frac{1}{\overline{T}(d_E)} \sum_{k=1}^{N} \overline{A}(k)\overline{W}(k) \sum_{m=1}^{k} Pr\left(d_E|d_H = k, m\right) Pr\left(m|d_H = k\right), \tag{12}$$

where $\overline{W}(k)$ is the average input weight for codewords of Hamming weight $d_H = k$.

2.2 Precoded Dicode

2.2.1 Error Event Distance Properties

Fig. 3 shows a trellis section for the dicode channel with precoder $g(D) = 1/(1 \oplus D)$. The branch labels are of the form c_i/x_i, where c_i is the input to the precoder at time i, and x_i is the corresponding channel output.

Referring to Fig. 3, it can be seen that an error word f may be decomposed into a sequence of $m = \lceil d_H(f)/2 \rceil$ simple error sub-events f_i, $i = 1, \ldots, m$. For $1 \leq i \leq m-1$, each sub-event is closed, Sub-event f_m may be either closed or open. The length of the sub-event f_i is denoted l_i, and the Hamming weight of a sub-event satisfies

$$d_H(f_i) = \begin{cases} 2 & i = 1, \ldots, m-1 \\ 2 & i = m \text{ and } d_H(f) \text{ even} \\ 1 & i = m \text{ and } d_H(f) \text{ odd.} \end{cases} \tag{13}$$

Let j_i^0 denote the bit position in the word where error event f_i begins. For closed events, let j_i^1 denote the bit position where it terminates. Then $l_i = j_i^1 - j_i^0 + 1$ for all closed sub-events, If f_m is open, we define $j_m^1 = N + 1$, and $l_m = j_m^1 - j_m^0$. Finally we define $L = \sum_{i=1}^{m} l_i$.

For an error event f_i, the total contribution $d_E^2(f_i)$ to the squared Euclidean distance at the channel output is given by

$$d_E^2(f_i) = d_H(f_i) + 4 \sum_{k=j_i^0+1}^{j_i^1-1} c_k. \tag{14}$$

The error word f has total squared Euclidean distance

$$d_E^2(f) = \sum_{i=1}^{m} d_E^2(f_i) = d_H(f) + 4 \sum_{i=1}^{m} \sum_{k=j_i^0+1}^{j_i^1-1} c_k. \tag{15}$$

We now invoke the assumption regarding the distribution of code bit values in the error events – namely, that their values are i.i.d. and equiprobable – to obtain an approximate contribution of an error word f of Hamming weight d to the average Euclidean distance spectrum. Under this assumption, we obtain

$$Pr(d_E^2(f) = z | d_H(f) = d, L) = \binom{L-d}{(z-d)/4} 0.5^{L-d}. \tag{16}$$

The i.i.d. approximation is supported for error words $\mathbf{f}$ with a small value of L by the action of the uniform interleaver. For error words with large value of L, the contribution to the dominant terms of the Euclidean error weight enumerator will be negligible, due to the low probability of generating small Euclidean distance.

2.2.2 Sub-Event Distribution at the Interleaver Output

Let $\mathbf{e}$ be an error word of Hamming weight d. A permuted error word $\mathbf{f}$ can be decomposed into $m = \lceil d/2 \rceil$ error events f_i, as described in Section 2.2.1. In this section, we determine the distribution of the total length L of error words generated by the action of a uniform interleaver upon the error word $\mathbf{e}$.

The distribution is computed in two steps. First, we find the number of unique back-to-back concatenations of m sub-events of total length L. Then, we determine the number of configurations in which the m sub-events can occur in a word of length N.

Consider the following description of the permuted error word f,

$$0, \ldots, 0, 1_1, 0_1, \ldots, 0_1, 1_1, 0, \ldots, 0, \ldots, 1_m, 0_m, \ldots, 0_m, 1_m, 0, \ldots, 0,$$

where the subscript denotes to which sub-event a bit belongs. There are $\binom{L-\lceil (d-1)/2\rceil -1}{\lfloor (d-1)/2\rfloor}$ unique back-to-back concatenations of sub-events f_i of total length L. If d is even, the remaining $N-L$ bits can be partitioned in $\binom{N-L+m}{m}$ different ways. If d is odd, the permutation has to end with an open error event, so there are $\binom{N-L+m-1}{m-1}$ possible permutations.

The conditional distribution $Pr(L|d)$ of the total length L, given an error word of Hamming weight $d_H(e)=d$, is therefore given by

$$Pr(L|d) = \frac{\binom{N-L+\lfloor d/2\rfloor}{\lfloor d/2\rfloor}\binom{L-1-\lceil (d-1)/2\rceil}{\lfloor (d-1)/2\rfloor}}{\binom{N}{d}}. \tag{17}$$

2.2.3 Approximation of the Euclidean Weight Enumerator

The approximate Euclidean weight enumerator $\tau(d_E)$ can be computed by substituting (16) and (17) into

$$\overline{T}(d_E) = \sum_{k=1}^{N} \overline{A}(k) \sum_{L=k}^{N-k} Pr\left(d_E|k, L\right) Pr\left(L|k\right). \tag{18}$$

In a similar way, the approximate average input error weight enumerator may be obtained from

$$\overline{w}(d_E) = \frac{1}{\overline{T}(d_E)} \sum_{k=1}^{N} \overline{A}(k)\overline{W}(k) \sum_{L=k}^{N-k} Pr\left(d_E|k, L\right) Pr\left(L|k\right). \tag{19}$$

3 Computed Bounds and Simulation Results

In this section, we compute truncated maximum likelihood (ML) union bound estimates using the method outlined in the paper, and we compare these with computer simulation results. We consider two outer encoders: (1) a rate 1/2, recursive systematic convolutional (RSC) encoder with encoder polynomials $(31, 33)_{octal}$, with parity bits punctured to yield code rate 8/9; and (2) a rate 4/5 turbo code consisting of a parallel concatenation of two of the RSC encoders. Both encoders use an information block of size $K = 4096$. The decoding technique used in the simulations is iterative, with *a posteriori* probability (APP) decoders used for the channel detectors as well as the decoders. Soft information is shared between all decoders for ten full iterations.

Fig. 4 shows the word error rate (WER) results for the rate 8/9 system, with and without precoder. Fig. 5 shows the corresponding results for bit error rate (BER).

In Table 1, the Euclidean weight enumerator estimates for the two systems are shown. (The authors are thankful to Dr. Douglas N. Rowitch [13] for providing us with weight enumerators.) For the dicode channel, the dominant contributor to the estimated error rate is the $d_E^2 = 4$ component; with the precoder, it is the $d_E^2 = 2$ component.

The interleavers used in the simulations and the uniform interleaver induce different weight enumerators. Therefore the estimated bounds and the simulation results differ and in the case without precoder the simulation curve crosses the bound curve. Fig. 6 compares the BER simulation results with the estimates obtained by applying our analytical approach to the specific interleaver. The fit between the analysis and simulation is improved, particularly in the precoded case. Without the precoder, accounting for the

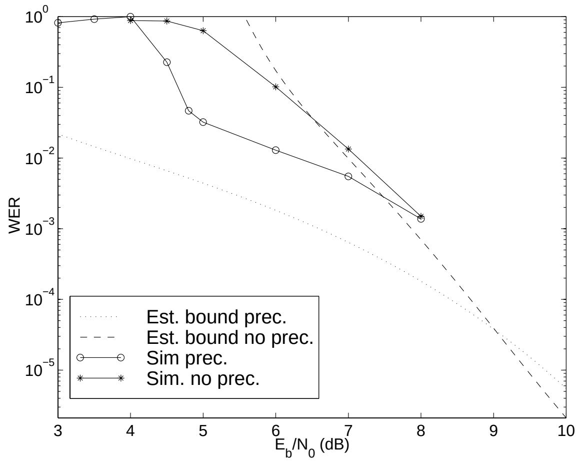

Figure 4: Word error rate union bound estimates and simulation results.

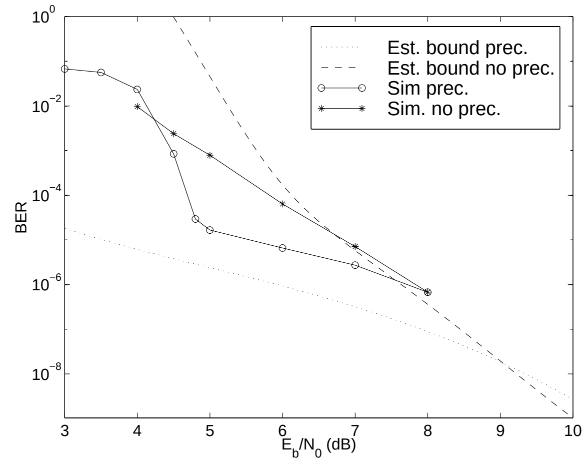

Figure 5: Bit error rate union bound estimates and simulation results.

Outer Code			Precoded	Not Precoded
d_H	$\overline{A}(d_H)$	d_E^2	$\tau(d_E)$	$\tau(d_E)$
2	510	2	0.4426	0.1122
3	21421	3	0.02421	0.2274
4	357864	4	0.8084	523.6
5	13192299	5	0.06468	14.25
6	389079383	6	4.255	21864
7	9010184299	7	0.3166	336.1
8	236369355044	8	15.68	386587

Table 1: Hamming and approximate Euclidean weight enumerators for systems with outer convolutional code.

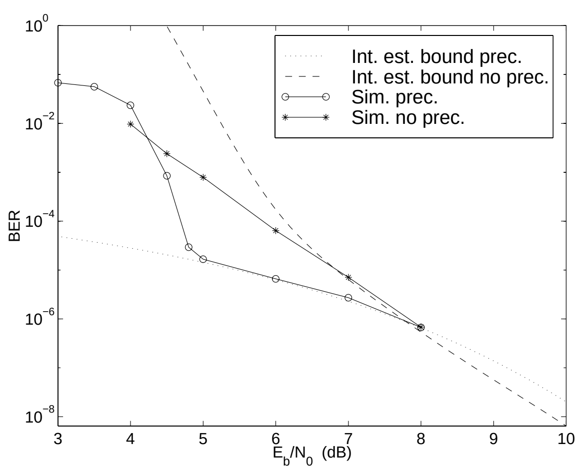

Figure 6: Truncated bound estimates for interleaver used vs. simulation result.

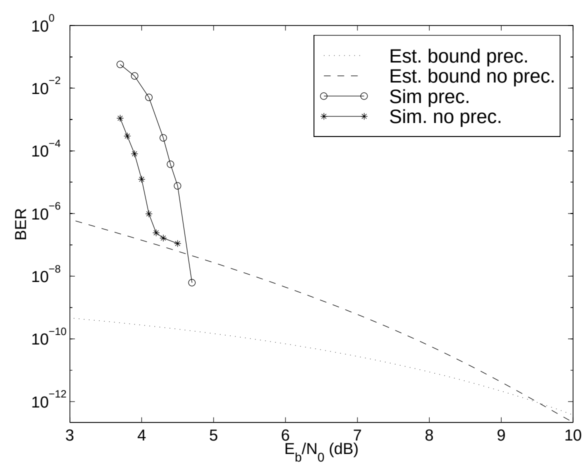

Figure 7: Bit error rate bound estimates vs. simulation results for precoded and non-precoded turbo system.

Interleaver	1	2	3	4
BER (6.0 dB) $/10^{-5}$	0.6610	0.2689	0.0766	0.0407

Table 2: BER with precoder at $E_b/N_0 = 6.0$ dB for three interleavers.

Outer Code			Precoded	Not Precoded
d_H	$\overline{A}(d_H)$	d_E^2	$\tau(d_E)$	$\tau(d_E)$
2	0.031	2	0.000024	0.000006
3	0.462	3	0.000000	0.000000
4	2.111	4	0.000004	0.031273
5	4.100	5	0.000000	0.000000
6	8.842	6	0.000024	0.464084
7	20.337	7	0.000001	0.000000
8	50.743	8	0.000008	2.114827

Table 3: Approximate Hamming and Euclidean weight enumerators for turbo coded systems.

specific interleaver used in the simulation made only a minor difference in the dominant term $\overline{T}(d_E^2 = 4)$. For smaller Euclidean distances, the relative changes were larger; for example, for the interleaver used in the simulations, the estimated weight enumerator for $d_E^2 = 2$ was 10 times larger than the average.

In contrast, with the precoder, the interleaver choice has a significant impact upon the estimate of the dominant term $\overline{T}(d_E^2 = 2)$. For the interleaver used, we computed the contribution made to the estimated Euclidean weight enumerator by error events of length $L \leq 15$ corresponding to error words of Hamming weight $d_H = 2$. The result was approximately 10 times the contribution of the average interleaver, leading to substantial improvement in the accuracy of the estimated error rate.

The impact of the interleaver in the precoded case is further reflected in simulation results for three additional interleavers. Table 2 shows their BER values at $E_b/N_0 = 6.0$ dB. The table suggests that suitable interleaver design can significantly improve the system performance.

Fig. 7 shows analytical BER estimates and simulation results for the rate 4/5 turbo coded systems. The Hamming weight enumerator and the estimated Euclidean weight enumerator for the turbo-coded system are shown in Table 3.

The bound for the precoded system is much lower at SNR up to about $E_B/N_0 = 9.5$ dB. However, in simulations, the system without precoder is superior down to $P_b \approx 2 \cdot 10^{-7}$, at which point the simulated BER curve flattens out and tends to follow the analytical curve. In fact, above 4.7 dB, the precoded system becomes superior to system without the precoder, as predicted by the analysis. The explanation for the behavior observed at very low SNR remains an open issue.

4 Conclusions

We have presented an analytical method for estimating the average Euclidean distance spectrum for a serial concatenated, trellis-coded partial response channel. The technique was applied to the dicode channel, with and without precoding. Using truncated union bounds, we derived analytical BER and WER results and compared them to computer simulations.

Future research directions are to bound the effect of the i.i.d. assumption, develop methods for higher order channels, and include the entire Hamming distance spectrum in the computations.

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