Distance-Enhancing Codes for Digital Recording

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***Abstract*— We review recent progress in the design of high-rate, distance-enhancing codes that achieve the matched-filter-bound (MFB) on the E^{2}PR4 channel, $h(D) = (1-D)(1+D)^3$, including a rate 8/9 block code satisfying a time-varying maximum-transition-run (TMTR) constraint. We then prove that these TMTR codes also achieve the MFB on the E^{3}PR4 channel, $h(D) = (1-D)(1+D)^4$. Finally, we describe a TMTR constraint that achieves the MFB on the PR2 channel, $h(D) = (1+D)^2$, and the EPR2 channel, $h(D) = (1+D)^3$, both of which are of interest as channel models in optical recording, and we present a new, rate 3/4 block code that satisfies this constraint. Computer simulation results confirm that the codes provide substantial performance improvement in additive, white, Gaussian noise (AWGN).**

I. Introduction

During the past few years, significant progress has been made in the design of high-rate distance-enhancing codes for magnetic recording channels equalized to high-order, extended partial-response (PR) targets, most notably the E^{2}PR4 channel with transfer polynomial $h(D) = (1-D)(1+D)^3$.

Behrens and Armstrong [1] first observed that, on the E^{2}PR4 channel, the rate 2/3, $(d,k) = (1,7)$ code provides a 2.2 dB increase in minimum squared-Euclidean distance, which we refer to as *coding gain*, that could be realized by using a detector matched to the $d = 1$ constraint, thus achieving the matched-filter-bound (MFB) of the channel. Karabed and Siegel [2] used a characterization of low-distance error events on the E^{2}PR4 channel to identify constraints that could support higher-rate codes, while achieving the same coding gain as the $d = 1$ constraint. These constraints, denoted $X_{\mathcal{F}}$, were defined by specifying a "forbidden list" $\mathcal{F}$ of inadmissible, finite-length code sequences. Examples included the NRZ $X_{\{101\}}$ constraint with capacity $C \approx 0.8113$, for which a rate 4/5 sliding-block code was constructed and evaluated by computer simulation. Soljanin [3] independently used error-event characterization to identify constraints that improve off-track performance in PR4, EPR4, and E^{2}PR4 channels, including the NRZ $X_{\{1010,0101\}}$ constraint with capacity $C \approx 0.8791$. The NRZ $X_{\{1010,0101\}}$ constraint is equivalent to the NRZI $X_{\{111\}}$ constraint, which limits transition runlengths to at most 2 and which supports a rate 4/5 NRZI block code that is simply the bitwise complement of the rate 4/5 Group Code Recording (GCR), $(d,k) = (0,2)$ code. Moon [4] independently discovered and investigated the performance of this NRZI constraint, as well as the specific rate 4/5 block code, and introduced the name "maximum transition run" (MTR) codes to describe the family of codes supported by the constraint. These authors simulated the performance of these distance-enhancing codes, considering a range of code rates and different detection methods. In particular, the expected 2.2 dB signal-to-noise ratio (SNR) improvement on the E^{2}PR4 channel with additive, white, Gaussian noise (AWGN), using Viterbi detection, was confirmed. Simulation results in [4] showed a substantial gain on the E^{3}PR4 channel, as well.

The aforementioned results have stimulated further investigations into error-event characterization [5], [6] and the design of higher-rate, distance-enhancing codes for PR channels [7],[8],[9],[10]. There has been particular interest in codes with rate 8/9 or above, which none of the previously mentioned constraints could support.

In this paper, we will describe recent progress in the determination of distance-enhancing constraints that achieve the MFB on the E^{2}PR4 channel and permit code rates 8/9 and higher. As shown in [8], there are "forbidden list" constraints that support a rate 8/9 code and provide a 2.2 dB coding gain on the E^{2}PR4 channel with a detector trellis matched to the constraint. For example, the NRZI constraint $X_{\mathcal{F}}$, with forbidden list $\mathcal{F} = \{1111, 11100\}$, has capacity $C \approx 0.9132$ and supports a rate 8/9, sliding-block code with moderate complexity, requiring 18 states in the trellis detector [8].

We will concentrate here upon a class of constraints with "time-varying" forbidden lists that have been found and investigated independently by Bliss [9], Fitzpatrick and Modlin [10], and Soljanin, *et al.* [8]. These constraints may be regarded as time-varying MTR constraints (TMTR), in which the maximum allowable runlength of 1's is a periodic function of the time at which the run begins. (Other trellis codes with time-varying constraints have also been proposed by Fredrickson [11].)

After introducing some notation and terminology in

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Section II, we consider in Section III a $k_1^{odd} = 3$, $k_1^{even} = 2$ constraint, in which a runlength of 1's beginning at an odd time index cannot exceed 3, while a runlength of 1's beginning at an even time index cannot exceed 2. This constraint provides 2.2 dB gain on E^{2}PR4 with a time-varying 16-state detector trellis. It has capacity $C \approx 0.916$ and can support a rate 8/9 block code, examples of which have been independently discovered and discussed in [9], [10], and [8]. (Time-varying constrained codes that achieve the MFB with rates exceeding 8/9 also are described in [10].) We will present simulation results that confirm a 2.2 dB SNR improvement at a channel bit error probability of approximately 10^{-5} when such TMTR constrained codes are used on the E^{2}PR4 channel in AWGN.

In Section IV we present a characterization of closed error events on the E^{3}PR4 channel with distance less than d_{MFB}. We use this to show that the codes based upon the $k_1^{odd} = 3$, $k_1^{even} = 2$ TMTR constraint, although designed for the E^{2}PR4 channel, also achieve the MFB on E^{3}PR4, increasing the minimum squared-Euclidean distance from $d^2 = 12$ to $d^2_{MFB} = 28$. We present simulation results that demonstrate an SNR improvement of 3.2 dB at a channel bit error probability of approximately 10^{-5} for a particular rate 8/9 TMTR block code on the E^{3}PR4 channel in AWGN.

Finally, in Section V we consider new distance enhancing constraints for the PR2 channel, $h(D) = (1+D)^2$, and the EPR2 channel, $h(D) = (1+D)^3$. These PR channels represent useful models for certain magnetic tape and optical recording systems. The minimum-distance of the PR2 channel falls 1.8 dB below the channel MFB. Wood [13] has shown that the $d = 1$ NRZI constraint, and its embodiment in the rate 2/3 (1,7) code, can be used to reach the MFB. We consider a new TMTR constraint, $k_1^{odd} = 2$, $k_1^{even} = 1$, that supports a rate 3/4 block code, while providing the same coding gain as the $d = 1$ constraint on the PR2 channel. The same TMTR constraint and code are then shown to provide a 3 dB coding gain on the EPR2 channel.

II. Preliminaries: Channel Models and Distance Properties

Digital magnetic recording channels at channel bit densities in the range $PW50/T_{ch} \in [2.0, 3.0]$ are often modeled by the discrete-time linear filters with impulse responses $h(D) = (1-D)(1+D)^3$ or $h(D) = (1-D)(1+D)^4$, denoted E^{2}PR4 and E^{3}PR4, respectively. For input sequences $x_1(D)$ and $x_2(D)$ over the binary alphabet $\{0, 1\}$, we define the corresponding input error sequence $e_x(D) = x_1(D) - x_2(D)$ over the alphabet $\{0, \pm 1\}$. (For convenience, we will occasionally denote $+1$ by $+$, and -1 by $-$.) The resulting output error sequence generated by the channel is given by $e_y(D) = h(D)e_x(D)$.

We say that the input error sequence $e_x(D)$ represents a *closed* error event if there exist finite integers $k_1 \le k_2$ such that $e_{x,k} = 0$, for $k < k_1$ and $k > k_2$, with $e_{x,k} \neq 0$, for $k = k_1, k_2$. A closed event is *simple* if there is no index $k_1 \le k \le k_2 - \nu$ such that $e_{x,k} = e_{x,k+1} = \cdots e_{x,k+\nu-1} = 0$, where ν is the degree of the channel polynomial $h(D)$.

TABLE I

Closed error events for the E^{2}PR4 channel, $h(D) = (1-D)(1+D)^3$.

d^2	e_x
6	+-+0000
8	+-+00+-+0000
	+-+-(+-)0000
	+-+-+(-+)0000

The closed event has squared-Euclidean distance

$$d^2(e_x) = \|e_y(D)\|^2.$$

We define the minimum (merged) distance d^2_{min} of the channel $h(D)$ to be

$$d^2_{min} = \min_{e_x(D)\, simple, closed} d^2(e_x).$$

As is well-known, this quantity governs the performance of the channel $h(D)$ in the presence of AWGN at moderate-to-high SNR.

We use the terminology *matched-filter-bound (MFB)* and the notation d^2_{MFB} to denote the energy in the channel impulse response, corresponding to the event $e_x = 1$. That is,

$$d^2_{MFB} = d^2(1) = \|h(D)\|^2,$$

and we say that a channel *achieves the MFB* if

$$d^2_{min} = d^2_{MFB}.$$

III. Rate 8/9 Distance-Enhancing Codes for E^{2}PR4

Table I shows, up to sign, the closed error events $e_x = e_{x,0}\, e_{x,1} \ldots$ on the E^{2}PR4 channel with $d^2(e_x) < d^2_{MFB} = 10$. In the table entries, a string enclosed in parentheses may appear any non-negative number of times.

It has been shown in [9], [10], [8] that a certain time-varying constraint on runlengths of consecutive 1's in an NRZI sequence can increase the minimum distance to the MFB on the E^{2}PR4 channel. Specifically, the constraint $k_1^{odd} = 3$, $k_1^{even} = 2$, which limits to 3 the maximum runlength of 1's beginning at an odd time index and limits to 2 the maximum runlength of 1's beginning at an even time index, eliminates all of the closed error events listed above. To see this, note that in any pair of binary NRZ sequences corresponding to an input error sequence beginning with the string $0+-+-$, where a leading 0 is explicitly incorporated, one sequence must contain a run of 4 consecutive transitions, as shown in bold below:

01010	11010
00101	**10101**.

The remaining input error events begin with the string $0+-+0$, where, again, a leading 0 is incorporated. The corresponding pairs of binary NRZ input sequences are given by:

01010 **01011** **11010** 11011
00100 **00101** **10100** **10101**

As indicated in bold, each event includes either a sequence with 4 consecutive transitions or a sequence with 3 consecutive transitions starting at an even time index.

It follows that this TMTR constraint achieves the MFB on the E^2PR4 channel. As shown in [9], [10], [8], there are 267 freely-concatenable, length-9 constrained words from which one can select any subset of 256 codewords for a rate 8/9 block code. Fig. 1 shows the simulated bit-error-rate (BER) performance of such a rate 8/9 code on the E^2PR4 channel in AWGN.

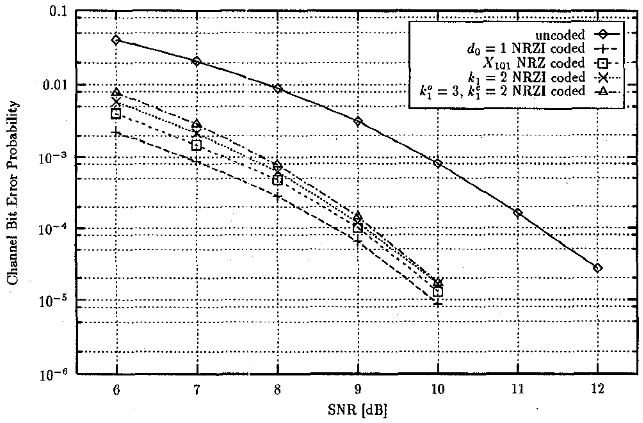

Fig. 1. Performance of uncoded and coded E^2PR4 systems.

Architectural issues relating to the VLSI implementation of a coded E^2PR4 channel based upon this rate 8/9, TMTR block code have been recently addressed in [12].

IV. Application of TMTR Codes to E^3PR4

We now show that the $k_1^{odd} = 3$, $k_1^{even} = 2$ TMTR constraint, which achieved the MFB on E^2PR4, also increases the minimum distance up to the MFB when used on the E^3PR4 channel. To do so, we must prove that all closed error events e_x on the E^3PR4 channel with squared-distance $12 \leq d^2(e_x) < 28 = d^2_{MFB}$ are eliminated by this constraint.

In [8], a partial characterization of closed error events with $12 \leq d^2(e_x) < 28$ was derived analytically. These events were shown to contain the string $+-+$ or $-+-$. It follows immediately from this that both the $d = 1$ and the $k_1 = 2$ MTR constraints eliminate all of these events and, therefore, they increase the minimum distance to at least the MFB. However, a more complete characterization of these events is required to obtain the corresponding result for the TMTR constraint.

TABLE II
Closed error events for E^3PR4 channel $h(D) = (1 - D)(1 + D)^4$

d^2	e_x
12	+-+00000
16	+-+00+-+00000
	+-+000+-+00000
20	+-+-+(-+)00000
	+-+-+-(+-)00000
	+-+0-+-00000
	+-+00+00+-+00000
	+-+00+-+00+-+00000
	+-+000+-+00+-+00000
	+-+000+-+000+-+00000
22	+-+-00000
	+-+0000+-+00000
	+-+00+00+00+-+00000
24	+00+-+00000
	+-+00+-+-+(-+)00000
	+-+00+-+-+-(+-)00000
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	+-+000+-+00+-+000+-+00000
	+-+00+00+00+00+-+00000
	+-+000+-+000+-+000+-+00000
26	+-0+-+00000
	+-+0-+-+-(+-)00000
	+-+0-+-+-+(-+)00000
	+-+00+-+-00000
	+-+0-+0-+-00000
	+-+000+-+-00000
	+00+00+-+00000
	+-+0000-+-00000
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	+-+000+-+00+00+00+-+00000
	+-+00+00+00+00+00+-+00000

Table II lists all input error sequences, up to sign and order reversal of the symbols, corresponding to simple, closed error events with $d^2(e_x) < 28$.

Examination of the list yields the stronger condition that these input error events, possibly with a leading 0 added, must contain one of the following strings, up to sign:

a) $0 + - + 0$
b) $+ - + - 0$
c) $+ - + - +$.

For case a), we have the NRZ input pairs:

01010 **01011** **11010** 11011
00100 **00101** **10100** **10101**

As indicated in bold, each event includes either a sequence with 4 consecutive transitions or a sequence with 3 consecutive transitions starting at an even time index.

Case b) corresponds to the following pairs:

$$\begin{matrix} 10100 & \mathbf{10101} \\ \mathbf{01010} & 01011 \end{matrix}$$

in each of which one component sequence contains 4 consecutive transitions, shown in bold. Similarly, in case c), it is clear that both component binary strings contain a run of 4 consecutive transitions.

The $k_1^{odd} = 3$, $k_1^{even} = 2$ TMTR constraint forbids any length-4 runs of transitions, as well as length-3 runs starting at even times; cases a), b) and c) are therefore eliminated. This confirms that the $k_1^{odd} = 3$, $k_1^{even} = 2$ TMTR constraint, and the codes derived from it, enhance the minimum distance of the E^3PR4 channel up to at least the MFB.

Fig. 2 shows the simulated performance on the E^3PR4 channel with AWGN of the rate 8/9, TMTR block code discussed above. The discrepancy between the measured SNR improvement and the coding gain is due to the error-event multiplicities of the simulated systems.

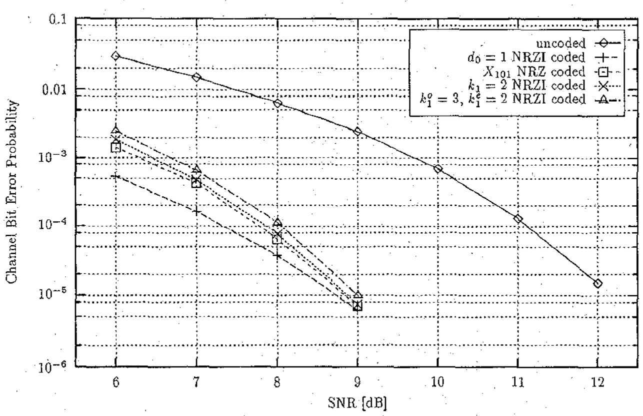

Fig. 2. Performance of uncoded and coded E^3PR4 systems.

V. A TMTR Code for PR2 and EPR2 Channels

The PR2 and EPR2 channels, with system polynomials $h(D) = (1+D)^2$ and $h(D) = (1+D)^3$, respectively, represent good models of certain high-density magnetic tape and optical recording systems. In this section, we verify the distance-enhancing properties of the $d = 1$ NRZI constraint on both of these channels. We then describe a new TMTR constraint that achieves the same coding gain on both channels, while permitting a 12.5% higher code rate. Simulation results for the rate 2/3 (1,7) code and a specific rate 3/4 block code confirm substantial SNR improvements relative to the uncoded channels.

A. NRZI $d = 1$ constraints for PR2 and EPR2

The PR2 channel, with binary inputs, has minimum distance $d_{min}^2 = 4$, with $d_{MFB}^2 = 6$. The closed error

TABLE III

Closed error events for the PR2 channel, $h(D) = (1+D)^2$.

d^2	e_x
4	+-(+-)00 +-+(-+)00

events with $d^2(e_x) = 4$ are shown, up to sign, in Table III.

To verify that the $d = 1$ constraint, which forbids the NRZ binary strings $\{101, 010\}$, eliminates all of these events, we first consider the events that contain the pattern $+ - +$. The component binary sequences of such an event clearly must contain either the string 101 or 010. The only error event that does not fall into this class is the event beginning with $+ - 0$. The pairs of component binary NRZ sequences must be of the form:

$$\begin{matrix} \mathbf{101} & \text{or} & 100 \\ 011 & & \mathbf{010} \end{matrix}$$

and, as highlighted in bold, one of the forbidden strings must occur in a component sequence of each pair. This confirms the distance-enhancing effect of the $d = 1$ constraint.

We remark that this gain was first brought to our attention by R. Hutchins and G. Sonu of IBM Corporation, Tucson. Wood [13] studied the use of a "modified linear canceller" to simplify the detection of $d = 1$ constrained codes on the PR2 channel and to achieve this 1.8 dB distance enhancement. We also remark that the Even-Mark-Modulation (EMM) code and, more generally, the family of matched-spectral-null (MSN) codes, arose from efforts to understand the coding gain provided by the rate 2/3 (1,7) code on the PR2 channel [14].

The EPR2 channel, with binary inputs, has minimum distance $d_{min}^2 = 10$, with $d_{MFB}^2 = 20$. Table IV shows the EPR2 closed error events, up to sign and order reversal of the symbols, with $10 \le d^2(e_x) < 20$. It is clear from the table of error events that the same line of reasoning as was applied in the case of PR2 proves that the $d = 1$ NRZI constraint prevents the occurrence of all of the closed error events with distance $d^2(e_x) < 20$. The coding gain is therefore $10\log_{10}(20/10) = 3$ dB.

For the $d = 1$ constrained PR2 (respectively, $d = 1$ constrained EPR2) system, one can use a Viterbi detector with a trellis requiring no more than 4 (respectively, 6) states. On both the PR2 and EPR2 channels, the only non-zero error input sequences that produce the all-0's output are the alternating sequences $\ldots + - + - \ldots$ and $\ldots - + - + \ldots$. Since the $d = 1$ NRZI constraint is finite-type [8] and does not support either of these error input sequences, it follows that the (1,7)-coded systems require only finite path memory.

B. TMTR constraints for PR2 and EPR2

The $d = 1$ NRZI constraint may also be interpreted as a MTR constraint with $k_1 = 1$. We relax this to a TMTR

TABLE IV

Closed error events for EPR2 channel $h(D) = (1+D)^3$

d^2	e_x
10	+-000
12	+-+ (-+) 000
	+-+- (+-) 000
	+-0+-000
14	+-0+-+(-+)000
	+-0+-+-(+-)000
	+-0+-0+-000
16	+-+(-+)0-+-(+-)000
	+-+(-+)0-+-+(-+)000
	+-+-(+-)0+-+(-+)000
	+-+-(+-)0+-+-(+-)000
	+-0+-0+-+(-+)000
	+-0+-0+-+-(+-)000
	+-0+-+(-+)0-+000
	+-0+-+-(+-)0+-000
	+-0+-0+-0+-000
18	+-00+-000
	+-0+-+(-+)0-+-(+-)000
	+-0+-+(-+)0-+-+(-+)000
	+-0+-+-(+-)0+-+(-+)000
	+-0+-+-(+-)0+-+-(+-)000
	+-+(-+)0-+0-+-(+-)000
	+-+(-+)0-+0-+-+(-+)000
	+-+-(+-)0+-0+-+-(+-)000
	+-0+-0+-0+-+(-+)000
	+-0+-0+-0+-+-(+-)000
	+-0+-0+-+(-+)0-+000
	+-0+-0+-+-(+-)0+-000
	+-0+-0+-0+-0+-000

constraint with parameters $k_1^{odd} = 2$ and $k_1^{even} = 1$. This constraint forbids 3 or more consecutive transitions, and therefore it eliminates all error input sequences with distance smaller than the MFB on the PR2 and EPR2 channels that begin with, up to sign, $+-+$. The only remaining input error sequences in Tables III and IV begin with, up to sign, $0+-0$, where a leading 0 is explicitly shown. The corresponding pairs of binary NRZ input sequences must take one of the following forms:

0100	**0101**	1100	**1101**
0010	0011	**1010**	**1011**

As indicated in bold, each pair contains either a sequence with 3 consecutive transitions, or a sequence with 2 consecutive transitions starting at an even time index. This confirms the distance-enhancing properties of the $k_1^{odd} = 2$, $k_1^{even} = 1$ TMTR constraint.

The NRZI TMTR constraint is generated by the labeled, period-2 graph H shown in Fig. 3. In the figure, states corresponding to even time index are represented by a square, while those corresponding to an odd time index are represented by a circle. The state names indicate the number of NRZI 1's since the most recent 0. Odd states 1 and 2 can be merged, after which so can even states 0 and 1. The resulting 3-state graph G, also shown in Fig. 3, is the Shannon graph [8] of the constraint.

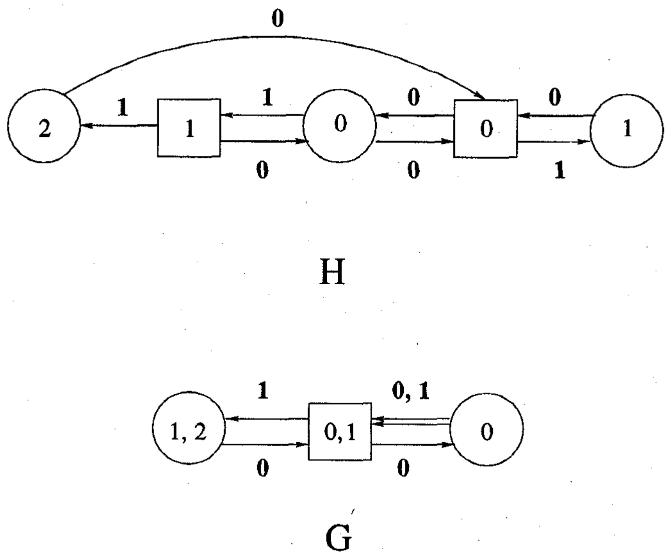

Fig. 3. Graph representations of NRZI $k_1^{odd} = 2$, $k_1^{even} = 1$ TMTR constraint.

The adjacency matrix of the graph G is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

and the capacity of this constrained system, given by the logarithm of the largest eigenvalue of the matrix A, is approximately $C \approx 0.7925$. The constraint therefore supports a code of rate $R = 3/4$.

The fourth power of the adjacency matrix is given by

$$A^4 = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 9 & 0 \\ 6 & 0 & 6 \end{bmatrix},$$

indicating that an NRZI rate 3/4 block code may be drawn from the set of 9 codewords of length 4 that begin and end at the state generated when even states 0 and 1 were merged.

From the list of candidate codewords, we exclude the all-0's word to aid in timing recovery and to ensure finite truncation depth in the detector. The remaining 8 codewords, listed in Table V, form the codebook for the block code. For the TMTR-constrained PR2 (respectively, TMTR-constrained EPR2) system, one can use a Viterbi detector with a time-varying trellis requiring no more than 4 (respectively, 6) states at any time.

Fig. 4 (respectively, Fig. 5) compares the simulated BER performance of the uncoded PR2 (respectively, uncoded EPR2) channel with the performance of the rate 2/3, (1,7)-coded PR2 (respectively, rate 2/3, (1,7)-coded EPR2) system and the rate 3/4, TMTR-coded PR2 (respectively, rate 3/4, TMTR-coded EPR2) system. As may be seen in the plots, the SNR improvement of the two coded systems exceeds the expected coding gain in this range of SNR. The discrepancy is due to the small error-event multiplicities of the coded channels.

TABLE V
Codebook for rate 3/4, $k_1^{odd} = 2$ and $k_1^{even} = 1$ TMTR code

Codeword
0001
0010
0100
0101
0110
1000
1001
1010

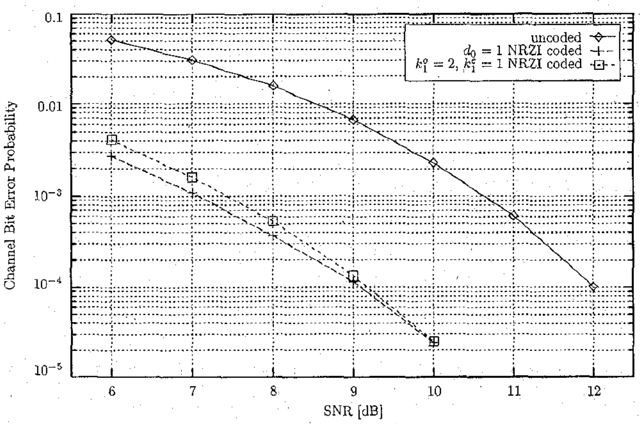

Fig. 4. Performance of uncoded and coded PR2 systems.

VI. Conclusions

This paper has described high-rate, distance-enhancing codes that achieve the matched-filter-bound (MFB) for several partial-response (PR) channels of interest in magnetic and optical recording. We reviewed a rate 8/9 block code satisfying a $k_1^{odd} = 3$, $k_1^{even} = 2$ time-varying maximum-transition-run (TMTR) constraint that achieves the MFB on the E^2PR4 channel, $h(D) = (1 - D)(1 + D)^3$, and E^3PR4 channel, $h(D) = (1 - D)(1 + D)^4$. We also described a $k_1^{odd} = 2$, $k_1^{even} = 1$ TMTR constraint that achieves the MFB on the PR2 channel, $h(D) = (1 + D)^2$ and the EPR2 channel, $h(D) = (1 + D)^3$. We also constructed a rate 3/4 block code that satisfies this constraint. Finally, we presented the results of performance simulations of these coded PR channels in AWGN, confirming the substantial SNR gains that the distance-enhancing codes can offer.

Note: After the presentation of this paper at TMRC'97, the authors learned that B. Brickner and J. Moon had independently observed the distance-enhancing properties of the $k_1^{odd} = 2$, $k_1^{even} = 1$ TMTR constraint on the PR2 channel and had designed a rate 3/4 block code using the same codebook as described in this paper. Their results appear in a University of Minnesota technical report dated July 10, 1997.

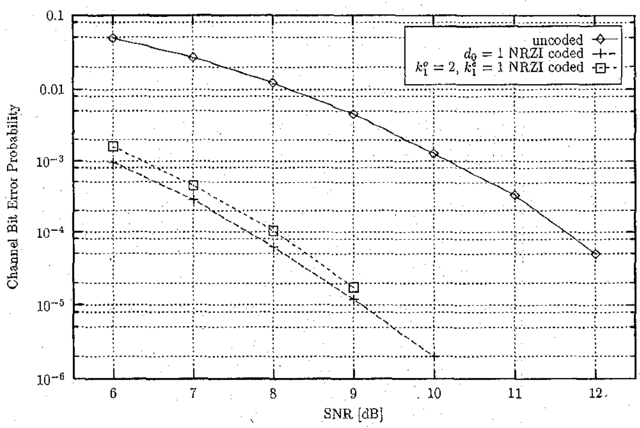

Fig. 5. Performance of uncoded and coded EPR2 systems.

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