Error Event Characterization and Coding for the Equalized Lorentzian Channel

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Abstract — **Coding methods to enhance the performance of sequence estimators for an intersymbol interference (ISI) channel are presented. The ISI channel considered is modeled with a Lorentzian step response, and equalized to a finite-duration, discrete impulse response. The coding follows from a characterization of the dominant error events for a suboptimal sequence estimator which uses a Euclidean distance metric for detection of a signal in colored noise.**

I. Introduction

The performance of a sequence detector for an ISI channel may be enhanced by eliminating the dominant error events via a constrained code that forbids a set of input sequences[1]. In [2], the dominant error events were characterized assuming the equalized signal has a white Gaussian noise component. In this paper, the problem of characterizing the dominant error events in colored noise is treated. The error event characterization motivates the construction of distance-enhancing codes.

II. Channel Model

Assume the input to the channel is a binary sequence, $a_n \in \{0,1\}$, and the detector receives a discrete sequence $r(D)$,

$$r(D) \quad = \quad a(D)x(D) + n(D)$$

where $x(D)$ is a finite duration target response and $n(D)$ is a zero mean, Gaussian random process with autocorrelation sequence $\phi_{nn,k}$. The detector chooses an estimate of the transmitted sequence closest to the received sequence in Euclidean distance, $\hat{a}(D) = \arg\min_{b(D)} \|r(D) - x(D)b(D)\|^2$. The probability of a symbol error for this estimate may be upper bounded by application of the union bound [3].

$$Pr(symbol\ error) \quad \leq \quad \sum_{\varepsilon_a(D)} w[\varepsilon_a(D)] Q\left(\frac{1}{2}\sqrt{\frac{(\boldsymbol{\varepsilon}_y^T\boldsymbol{\varepsilon}_y)^2}{\boldsymbol{\varepsilon}_y^T\boldsymbol{R}_{nn}\boldsymbol{\varepsilon}_y}}\right)$$

where $w[\varepsilon_a(D)]$ is a weighting factor for the event, $\varepsilon_y(D) = a(D)x(D) - \hat{a}(D)x(D) = \varepsilon_a(D)x(D)$, $Q(\cdot)$ is the error function, and $\boldsymbol{R}_{nn}$ is the autocorrelation matrix of $n(D)$. Define the squared argument of the Q function as the effective distance, d_{eff}^2.

III. Error Event Characterization

The performance of the sequence estimator at high SNR is determined by the error sequences $\boldsymbol{\varepsilon}_a$ with small effective distance. Using the relation

$$\{\boldsymbol{\varepsilon}_a | d_{eff}^2 \leq K_d\} \subseteq \{\boldsymbol{\varepsilon}_a | \boldsymbol{\varepsilon}_y^T\boldsymbol{\varepsilon}_y \leq K_d K_\lambda\},$$

where K_λ is an upper bound on $\frac{\boldsymbol{\varepsilon}_y^T\boldsymbol{R}_{nn}\boldsymbol{\varepsilon}_y}{\boldsymbol{\varepsilon}_y^T\boldsymbol{\varepsilon}_y}$, one can form the set $\{\boldsymbol{\varepsilon}_a | d_{eff}^2 \leq K_d\}$ by searching over all sequences with $\boldsymbol{\varepsilon}_y^T\boldsymbol{\varepsilon}_y \leq K_d K_\lambda$ and discarding sequences with $d_{eff}^2 \geq K_d$. A similar approach was described in [4].

The search was performed for a target response which has been proposed for use on the Lorentzian channel, $x_{EPR4}(D) = (1-D)(1+D)^2$. For a Lorentzian channel at density $\beta = 2.5$ equalized to the target x_{EPR4}, the dominant error event is $\boldsymbol{\varepsilon}_a = +-+(-+)000$, with $d_{eff}^2 = .36$.

IV. Coding to Improve Performance

We constrain the inputs to the channel to prevent minimum effective distance events. The constrained input, denoted $\mathsf{X}_{\mathcal{F}}$, is described by a set of forbidden strings $\mathcal{F} = \{\omega_1, \omega_2, \cdots\}$ over the input alphabet. For example, the constraint $\mathsf{X}_{\mathcal{F}=\{01010,10101,101000,010111\}}$, with capacity $C \approx .913$, eliminates the event $\boldsymbol{\varepsilon}_a = +-+(-+)000$, increasing the minimum effective distance to $d_{eff}^2 = .42$, corresponding to the event $\boldsymbol{\varepsilon}_a = +0(+0)00$. Applying the constraint to the input of the channel will yield an asymptotic coding gain $ACG = 10log_{10}(.36/.42) \approx .67dB$. Simulation results confirm that the coded system exhibits the expected gain at a symbol error rate of 10^{-5}.

V. Conclusion

The techniques presented may be used to form the set of dominant error events for a Euclidean distance metric sequence estimator with possibly colored noise at the input. An interesting result is that a constrained code which provides no coding gain in white noise may provide coding gain in colored noise.

References

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