A Comparison of Long Versus Short Spreading Sequences in Coded Asynchronous DS-CDMA Systems

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***Abstract*—The performance of turbo-coded asynchronous direct sequence code division multiple access (DS-CDMA) using long and short spreading sequences is compared by both analysis and simulation. For coded systems with a conventional matched filter (MF) receiver, three analytical methods with different complexity are compared: the standard Gaussian approximation, the improved Gaussian approximation (IGA), and the density function approach. It is shown that while the standard Gaussian approximation is fairly accurate for the long sequences, it is too optimistic for the short sequences. For the short-sequence systems, the IGA gives an accurate estimate for the performance with much less complexity than the density function approach. The analysis shows that for either the additive white Gaussian noise (AWGN) channel or the flat Rayleigh fading channel and a MF receiver, there is a degradation in the average performance of the turbo-coded short-sequence systems compared to the long-sequence systems due to the fact that the cross-correlations are not time-varying. However, the short-sequence systems are amenable to the use of an interference suppression technique designed to minimize the mean square error. Such a minimum mean square error (MMSE) receiver in the turbo-coded system is shown to outperform the long-sequence system with the MF receiver, especially when there is a near–far problem, as previously observed in a convolutionally-coded system. Finally, similar results are obtained by computer simulations for the turbo-coded CDMA systems on a frequency-selective Rayleigh fading channel.**

***Index Terms*—Channel coding, cochannel interference, code division multiple access (CDMA), land mobile radio cellular systems, spread spectrum communications.**

I. Introduction

IN THE RECENT past, there has been considerable interest in the subject of multiuser detection in direct-sequence code-division multiple-access (DS-CDMA) systems. One important type of multiuser receiver, the minimum mean-square-error (MMSE) detector [1], [2], relies on the cyclostationarity of the interference statistics and requires short spreading sequences. However, current cellular CDMA systems (e.g., IS-95) are using long spreading sequences together with the conventional matched filter (MF) receiver. With a period much larger than a data bit, the long sequences appear essentially random. That is, the statistics of the multiple-access interference (MAI) change randomly from bit to bit, and the performance is determined by the average interference level. In a short-sequence system with slowly varying delays, the cross-correlations between users remain unchanged over time, and some users may be trapped in a disadvantageous situation. Fortunately, the MMSE receiver can be used to improve the performance for the short-sequence systems. In [3], the distribution of the bit-error probability is obtained by a hybrid simulation/analysis approach for the short-sequence and long-sequence systems in a scenario of multiple cells and fading channels, either uncoded or with a convolutional code. It was found that a somewhat larger performance spread exists for short sequences than for long sequences when the MF receiver was used, but that the short-sequence MMSE receiver outperforms either system using a MF receiver. Recently, turbo coding has been adopted in wireless CDMA systems to improve the system quality and capacity [4], [5]. In this paper, the performance of a turbo-coded CDMA system using long and short spreading sequences is compared in a single cell scenario.

Several attempts have been made to evaluate the performance of DS-CDMA systems. The simplest method, called the standard Gaussian approximation (GA), models the MAI as a Gaussian noise process. The GA is fairly accurate for the MMSE detector [6] but less accurate for the MF receiver. To obtain more accurate results for the MF receiver, the density functions of the random MAI were found for both random and short sequences to evaluate the uncoded bit-error probability in [7] and [8]. A so-called "improved Gaussian approximation" (IGA) was presented in [9] and [10] for the random-sequence system to show the bit-to-bit-error dependence when a hard-decision block code is used. In this paper, both the density function and the IGA approaches are extended to a coded CDMA system with maximum-likelihood soft-decision decoding for both short sequences and long sequences. We show that, for an additive white Gaussian noise (AWGN) channel, the GA method is fairly accurate for long sequences but is too optimistic for short sequences. The IGA method achieves roughly the same accuracy as the density function approach but with much less computational complexity. In addition, the IGA method can be easily extended to a flat fading channel. Our analytical results show that with the MF receiver on either the AWGN channel or the flat Rayleigh fading channel, there is a significant degradation of the average performance of the

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turbo-coded system with the short sequences compared to that with the long sequences. However, the short-sequence system with the MMSE receiver outperforms either of the systems with the MF receiver, which agrees with the results found in [3] for either uncoded or convolutionally-coded CDMA systems. Finally, computer simulations are carried out for the turbo-coded systems on a frequency-selective Rayleigh fading channel, and it is shown that similar conclusions hold.

The paper is organized as follows. The system model and the MF receiver are described in Section II. In Section III, three analytical approaches to evaluate the performance are presented, together with the turbo coding scheme in our system. The MMSE receiver and performance comparison are provided in Section IV. Finally, conclusions are drawn in Section V.

II. System Model

We initially consider a coded binary direct-sequence CDMA system on the AWGN channel. The transmitted signal of the kth user is

$$s_k(t) = \sqrt{2P_k}a_k(t)b_k(t)\cos(\omega_c t + \phi_k) \tag{1}$$

where
- P_k transmitted power;
- ω_c carrier frequency;
- ϕ_k carrier phase.

The spreading waveform $a_k(t)$ is given by

$$a_k(t) = \sum_{n=-\infty}^{\infty} a_n^{(k)} p_{T_c}(t - nT_c) \tag{2}$$

where
- $\{a_n^{(k)}\}$ discrete signature sequence of elements from the set $\{-1, +1\}$;
- $p_\tau(t)$ rectangular pulse which starts at $t = 0$ and ends at $t = \tau$;
- T_c duration of a chip.

We assume that each code symbol is spread with N_s chips, i.e., $T_s = N_s T_c$, where T_s is the duration of a code symbol. For the system with short sequences, the period of each signature sequence is assumed to be N_s. If long spreading sequences are employed with periods much larger than N_s, $\{a_n^{(k)}\}$ is modeled as a sequence of independent identically distributed (i.i.d.) random variables such that $\Pr\left\{a_k^{(n)} = +1\right\} = \Pr\left\{a_k^{(n)} = -1\right\} = 1/2$.

The encoded signal $b_k(t)$ can be expressed as

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_i^{(k)} p_{T_s}(t - iT_s) \tag{3}$$

where $b_i^{(k)} \in \{\pm 1\}$ is the code symbol sequence generated by the encoder.

For an asynchronous system with K users, the received signal is

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} s_k(t - \tau_k) + n(t) \\ &= \sum_{k=1}^{K} \sqrt{2P_k}a_k(t-\tau_k)b_k(t-\tau_k)\cos(\omega_c t + \psi_k) + n(t) \end{aligned} \tag{4}$$

where τ_k accounts for propagation delay and lack of synchronism between transmitters, $\psi_k = \phi_k - \omega_c\tau_k$, and $n(t)$ is a white Gaussian noise process with two-sided spectral density $N_0/2$. Without loss of generality, index $k = 1$ is assigned to the desired user, and we assume $\tau_1 = 0$ and $\psi_1 = 0$. The decision statistic at the output of a coherent correlation receiver for the ith code symbol is given by [11]

$$U_i = T_s\sqrt{\frac{P_1}{2}}b_i^{(1)} + T_s\sum_{k=2}^{K}\sqrt{\frac{P_k}{2}}I_k(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i}) + \eta_i \tag{5}$$

where $I_k(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i})$ is the interference term from the kth user, and η_i is Gaussian with zero mean and variance $N_0T_s/4$. The vector $\mathbf{b}_{k,i} = \left(b_{i,-1}^{(k)}, b_{i,0}^{(k)}\right)$ represents a pair of consecutive code symbols for the kth user, and the vectors can be considered independent of each other for different values of i, by assuming the application of a channel interleaver with sufficient depth. We also assume that the $\{\tau_k\}$ are fixed for the entire codeword, while the $\{\psi_{k,i}\}$ change independently from bit-to-bit. For notational simplicity, the following normalized decision variable is introduced

$$U_i' = \frac{U_i}{T_c\sqrt{\frac{P_1}{2}}} = N_s b_i^{(1)} + \sum_{k=2}^{K}\sqrt{\frac{P_k}{P_1}}I_k'(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i}) + \eta_i' \tag{6}$$

where $I_k'(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i}) = N_s I_k(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i})$ and $\eta_i' = \eta_i/(T_c\sqrt{P_1/2})$.

A. System with Short Sequences

For the system with short sequences, the delay τ_k is assumed to be uniformly distributed in the interval $[0, N_sT_c)$, and can be written as $\tau_k = (p_k + \delta_k)T_c$, where $p_k = 0, 1, \ldots, N_s - 1$ with equal probability, and δ_k is uniform on $[0, 1)$. The normalized interference, $I_k'(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i})$, is defined as

$$I_k'(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i}) = W_k(\mathbf{b}_{k,i}, \tau_k)\cos\psi_{k,i} \tag{7}$$

where

$$W_k(\mathbf{b}_{k,i}, \tau_k) = \frac{1}{T_c}\left[b_{i,-1}^{(k)}R_k(\tau_k) + b_{i,0}^{(k)}\hat{R}_k(\tau_k)\right]. \tag{8}$$

The functions $R_k(\tau_k)$ and $\hat{R}_k(\tau_k)$ are the continuous-time partial cross-correlation functions, defined in [11].

Equivalently, $W_k(\mathbf{b}_{k,i}, \tau_k)$ can be represented as

$$W_k(\mathbf{b}_{k,i}, \tau_k) = \begin{cases} b_{i,0}^{(k)}\theta_k(\tau_k) & \text{if } b_{i,0}^{(k)} = b_{i,-1}^{(k)} \\ b_{i,0}^{(k)}\hat{\theta}_k(\tau_k) & \text{if } b_{i,0}^{(k)} \neq b_{i,-1}^{(k)} \end{cases} \tag{9}$$

where the functions $\theta(\tau_k)$ and $\hat{\theta}(\tau_k)$ are given by

$$\begin{aligned} \theta_k(\tau_k) &= \theta_{k,1}(p_k)(1 - \delta_k) + \theta_{k,1}(p_k + 1)\delta_k \\ \hat{\theta}_k(\tau_k) &= \hat{\theta}_{k,1}(p_k)(1 - \delta_k) + \hat{\theta}_{k,1}(p_k + 1)\delta_k \end{aligned} \tag{10}$$

and $\theta_{k,j}$ and $\hat{\theta}_{k,j}$ are the periodic (or even) and the odd cross-correlation functions for the spreading sequences of user k and user j, respectively [11].

B. System with Random Sequences

For the system with random spreading sequences, the normalized decision variable is shown to be [7]

$$U_i' = N_s b_i^{(1)} + \sum_{k=2}^{K}\sqrt{\frac{P_k}{P_1}}W_{k,i}(\delta_k)\cos\psi_{k,i} + \eta_i' \tag{11}$$

where

$$W_{k,i}(\delta_k) = L_{k,i}\delta_k + M_{k,i}(1-\delta_k) + X_{k,i} + Y_{k,i}(1-2\delta_k). \quad (12)$$

The random variables $L_{k,i}$ and $M_{k,i}$ are uniform on $\{-1, +1\}$, and $X_{k,i}$ and $Y_{k,i}$ have densities

$$p_{X_{k,i}}(j) = \binom{A}{\frac{j+A}{2}} 2^{-A};\ j \in \{-A, -A+2, \ldots, A-2, A\} \quad (13)$$

and

$$p_{Y_{k,i}}(j) = \binom{B}{\frac{j+B}{2}} 2^{-B};\ j \in \{-B, -B+2, \ldots, B-2, B\} \quad (14)$$

respectively.

The quantities A and B are related to $C = C_{1,1}(1)$, the discrete aperiodic autocorrelation of the signature sequence of user 1, offset by one chip, by [7]

$$A = \frac{N_s - 1 + C}{2} \quad (15)$$

and

$$B = \frac{N_s - 1 - C}{2}. \quad (16)$$

Note that the quantity B is the number of times the desired signal's spreading sequence changes state, either from -1 to $+1$ or from $+1$ to -1, within one bit duration.

For a random sequence, the random variable C has the density

$$p_c(j) = \binom{N_s - 1}{\frac{j+N_s-1}{2}} 2^{1-N_s}$$
$$j \in \{1-N, 3-N, \ldots, N-3, N-1\}. \quad (17)$$

Note that the random variables $L_{k,i}$, $M_{k,i}$, $X_{k,i}$ and $Y_{k,i}$ are independent of each other for any time i and any interfering user k, conditioned on the quantity C [7]. Also, the random variable W_{k_1,i_1} is independent of W_{k_2,i_2} for either $k_1 \neq k_2$ or $i_1 \neq i_2$, conditioned on C.

III. Performance Analysis

A. Gaussian Approximation

When the user number K is large, the standard GA [11] can be used to evaluate the average signal-to-interference-and-noise ratio (SINR) and bit-error probability. For the short-sequence system, the interference component I'_k is modeled as a Gaussian random variable with zero mean, and the variance is obtained by averaging over $\mathbf{b}_{k,i}$, τ_k and $\psi_{k,i}$ [11]:

$$\mathrm{Var}\{I'_k\} = \frac{1}{6N_s}[2\mu_{k,1}(0) + \mu_{k,1}(1)] \quad (18)$$

where

$$\mu_{k,j}(p) = \sum_{n=1-N_s}^{N_s-1} C_{k,j}(n)C_{k,j}(n+p) \quad (19)$$

and $C_{k,j}(n)$ is the discrete aperiodic cross-correlation function for the kth and jth signature sequences [11].

The average SINR is given as

$$\gamma_s = \frac{1}{\frac{N_0}{E_s} + \frac{2}{N_s^2}\sum_{k=2}^{K}\frac{P_k}{P_1}\mathrm{Var}\{I'_k\}}. \quad (20)$$

Invoking the standard union bound for the convolutional code, the bit-error probability is bounded by

$$P_e \leq \sum_{d=d_{\mathrm{free}}}^{\infty} \beta_d Q\left(\sqrt{2d\gamma_s}\right) \quad (21)$$

where $\{\beta_d\}$ is the distance spectrum of the code [12] and $Q(x) \triangleq \left(1/\sqrt{2\pi}\right)\int_x^{\infty} \mathrm{e}^{-t^2/2}dt$. As an approximation, the sum is truncated after the first ten terms in the following analysis. For random spreading sequences, the average SINR is given by [13]

$$\gamma_s = \frac{1}{\frac{N_0}{E_s} + \frac{2}{3N_s}\sum_{k=2}^{K}\frac{P_k}{P_1}}. \quad (22)$$

Although the GA is easy to evaluate, it may be optimistic in predicting the bit-error probability, especially for the short-sequence systems, as shown in Section III-C. More accurate methods are developed in the following two subsections to predict the performance and provide useful insights of the system.

B. Density Function Approach

Using numerical techniques, we can predict the performance of DS-CDMA based on the density function of the MAI. This approach was used to evaluate the error probabilities of uncoded systems with both long and short spreading sequences in [7], [8]. In this subsection, the density function approach is extended to coded CDMA systems with maximum-likelihood soft-decision decoding.

For the coded system with short spreading sequences, considering a correct trellis path and an erroneous path with a Hamming distance d, the metric difference is

$$M_d = \frac{1}{N_s}\sum_{i=1}^{d} b_i^{(1)} U'_i \quad (23)$$

where U'_i is the decision statistic defined in (6). The metric difference can be written as

$$M_d = d + \eta_d + Z \quad (24)$$

where η_d is Gaussian with variance $dN_0/2E_s$, and $E_s = T_s P_1$. The MAI Z is given by

$$Z = \frac{1}{N_s}\sum_{k=2}^{K}\frac{P_k}{P_1}\sum_{i=1}^{d} b_i^{(1)} W_k(\mathbf{b}_{k,i}, \tau_k)\cos\psi_{k,i}. \quad (25)$$

For a given delay τ_k, define $Z_{k,i}(\tau_k) = b_i^{(1)} W_k(\mathbf{b}_{k,i}, \tau_k)\cos\psi_{k,i}$. Note that $W_k(\mathbf{b}_{k,i}, \tau_k)$ is given in (9), and $b_i^{(1)}$ can be absorbed into $\cos\psi_{k,i}$. The conditional density function of $Z_{k,i}$ is given by [7]

$$f_{Z_{k,i}}(z; \tau_k) = \frac{1}{2}[q(z; \theta_k(\tau_k)) + q(z; \hat{\theta}_k(\tau_k))] \quad (26)$$

where the function $q(z;\theta)$ is defined by

$$q(z;\theta) = \frac{1}{\pi\theta\sqrt{\frac{1-z^2}{\theta^2}}}, \quad |z| < \theta. \quad (27)$$

Since the $Z_{k,i}$ are independent for different values of i, conditioned on τ_k, the conditional density function of the summation $Z_k(\tau_k) \triangleq \sum_{i=1}^{d} Z_{k,i}(\tau_k)$ is given by

$$f_{Z_k}(z;\tau_k) = f_{Z_{k,i}}(z;\tau_k) * f_{Z_{k,i}}(z;\tau_k) * \ldots * f_{Z_{k,i}}(z;\tau_k) \quad (28)$$

where, in (28), there are $d-1$ convolutions indicated by the asterisks.

Averaging over τ_k and taking scaling factors into consideration, the unconditional density function of $Z_k \triangleq E_{\tau_k}[(P_k/P_1N_s)Z_k(\tau_k)]$ is given by

$$f_{Z_k}(z) = \frac{P_1N_s}{P_k}\int_0^{N_sT_c} f_{Z_k}\left(\frac{P_1N_s}{P_k}z;\tau_k\right)d\tau_k \quad (29)$$

$$= \frac{P_1N_s}{P_k}\sum_{p=0}^{N_s-1}\int_0^{T_c} f_{Z_k}\left(\frac{P_1N_s}{P_k}z;pT_c+\delta_k\right)d\delta_k. \quad (30)$$

This averaging can be achieved by a simple numerical integration. Note that, although the MAI Z_k is dependent on the bit sequence $\left\{b_i^{(1)}\right\}$ of the desired user, the density function $f_{Z_k}(z)$ remains the same no matter which values of ± 1 the $\left\{b_i^{(1)}\right\}$ take, due to the symmetric distributions of $\left\{b_i^{(k)}\right\}$ and $\{\cos\psi_{k,i}\}$ about zero. So, the density of the total MAI can be obtained by

$$f_Z(z) = f_{Z_2}(z) * f_{Z_3}(z) * \ldots * f_{Z_K}(z). \quad (31)$$

With the density function of $f_Z(z)$, we can evaluate the pairwise error probability as

$$P_2(d) = \Pr[M_d < 0] = \int_{-\infty}^{\infty} Q\left((d+z)\sqrt{\frac{2E_s}{dN_0}}\right) f_Z(z)dz. \quad (32)$$

The final bit-error probability is bounded by

$$P_e \le \sum_{d=d_{\text{free}}}^{\infty} \beta_d P_2(d). \quad (33)$$

For the coded system with long spreading sequences, the interference components of different users are independent, conditioned on the quantity C (or B) of the desired spreading sequence [7]. Since the desired sequence changes randomly from bit-to-bit within a code trellis, obtaining the density function of the MAI component in the decoding metric difference is a prohibitively difficult task. However, the density function approach can be modified to give the performance of a user with a short spreading sequence in an environment where all interfering users are assumed to employ long spreading sequences. As shown in [14], in an uncoded CDMA system where the interfering spreading sequences are long (i.e., random) and asynchronous, the bit-error probability for the desired user using a fixed (short) sequence with $B = (N_s-1)/2$ is identical to that using the long sequences. We compared the simulation results (not presented) of a short sequence of the desired user with $B = (N_s-1)/2$ to those of a random desired sequence in a coded CDMA system, when all other users employ long sequences, and the two situations resulted in almost identical performance for the range of BER where the simulation was feasible.

Given the delay $\tau_k = \delta_k T_c$, the distribution of $W_{k,i}$ is determined by (12) with four discrete random variables, $L_{k,i}$, $M_{k,i}$, $X_{k,i}$ and $Y_{k,i}$. Its density function can be put into the form of a sum of impulses

$$f_{W_{k,i}}(w;\tau_k) = \sum_j f(w_j;\delta_k)\delta(w-w_j) \quad (34)$$

where $\{w_j\}$ are the possible values that $W_{k,i}$ can take, and $f(w_j;\delta_k)$ is the probability that $W_{k,i}$ takes the value of w_j, conditioned on the delay $\delta_k T_c$.

The conditional density function of $Z_{k,d}(\tau_k)$ is given by

$$f_{Z_{k,d}}(z;\tau_k) = \sum_j f(w_j;\delta_k)q(z;w_j) \quad (35)$$

where $q(z;w_j)$ is defined in (27). Following (28)–(33), the final bit-error probability can be obtained for the system in which only the desired user employs a short sequence.

In Fig. 1, the density functions of the total MAI in the soft decision metric, $f_Z(z)$, are shown for an error event with Hamming distance $d = 8$, the minimum distance of both the convolutional code and the turbo code used in this paper. The processing gain of the CDMA system is 31, with 12 users. It can be seen that the density function for the long-sequence system matches the Gaussian distribution for a broad range, while the density function for the short-sequence system diverges from the Gaussian shape much earlier. This explains why the GA is fairly accurate for long-sequence systems but gives optimistic results for the short-sequence systems. The bit-error probability obtained by this method is presented in the next subsection.

C. Improved Gaussian Approximation

In a coded system, the MAI components between neighboring code symbols are correlated, since the delays and, perhaps, the phases of the interfering signals are highly correlated throughout the codeword. In [9] and [10], the effect of correlation on packet throughput is shown for a DS-CDMA system with hard-decision block coding and random signature sequences. The approach in [9] and [10] is to obtain the probability density of the variance of the MAI, conditioned on $\{\tau_k\}$ and $\{\psi_k\}$, and the performance is evaluated by averaging over the density function. In this subsection, we extend this approach to a DS-CDMA system with short sequences and maximum-likelihood soft-decision decoding. In addition, we assume the only correlation factor is due to the relative delay. For example, for a DS-CDMA system with chip rate around 1 Mc/s, a simple calculation shows that the relative delays will remain within a fraction of a chip duration for thousands of bits, even for mobiles traveling at freeway speed. However, the phase difference changes relatively fast because of the motion and the frequency shift of the transmitted signal, and a block interleaver of moderate size can further remove the correlation of the relative phases. Thus, we assume the $\{\tau_k\}$ are fixed for

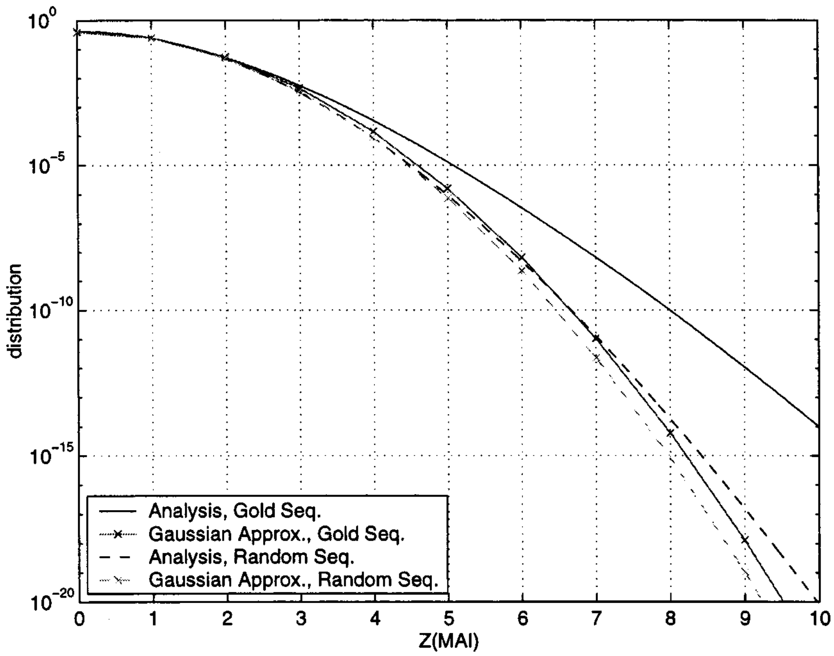

Fig. 1. Density functions of total MAI in soft decision metric for an error event with Hamming distance $d = 8$ of coded DS-CDMA system, AWGN channel, $N_s = 31$, $K = 12$, $PR = 0$ dB, $R_c = 1/2$. Gold sequences are used in the short-sequence system.

the entire codeword, while the $\{\psi_k\}$ change independently from bit to bit.

Given the delay τ_k (or equivalently, δ_k and p_k) in the short-sequence system, from (9) and (10), we obtain the conditional variance of W_k as

$$\begin{aligned} V_k \triangleq & \mathrm{Var}\{W_k|p_k,\delta_k\} \\ =& \frac{1}{2}[\Delta\theta_k(p_k)\delta_k + \theta_{k,1}(p_k)]^2 + \frac{1}{2}[\Delta\hat{\theta}_k(p_k)\delta_k + \hat{\theta}_{k,1}(p_k)]^2 \\ \triangleq & \alpha(\delta_k+\beta)^2 + \gamma \end{aligned} \tag{36}$$

where

$$\begin{aligned} \Delta\theta_k(p_k) =& \theta_{k,1}(p_k+1) - \theta_{k,1}(p_k) \\ \Delta\hat{\theta}_k(p_k) =& \hat{\theta}_{k,1}(p_k+1) - \hat{\theta}_{k,1}(p_k) \\ \alpha =& \frac{1}{2}\left[\Delta^2\theta_k(p_k) + \Delta^2\hat{\theta}_k(p_k)\right] \\ \beta =& \frac{\Delta\theta_k(p_k)\theta_{k,1}(p_k) + \Delta\hat{\theta}_k(p_k)\hat{\theta}_{k,1}(p_k)}{\Delta^2\theta_k(p_k) + \Delta^2\hat{\theta}_k(p_k)} \\ \text{and } \gamma =& \frac{[\Delta\theta_k(p_k)\hat{\theta}_{k,1}(p_k) - \Delta\hat{\theta}_k(p_k)\theta_{k,1}(p_k)]^2}{2\left[\Delta^2\theta_k(p_k) + \Delta^2\hat{\theta}_k(p_k)\right]}. \end{aligned}$$

Given the value of p_k, V_k is a function of the random variable δ_k, which is uniformly distributed over $[0,1)$. Applying the method for determining the statistics of a random variable $Y = aX^2$ in terms of the statistics of X [15], the conditional density of V_k is found to take three forms, according to different values of α and β:

1) if $\alpha = 0$

$$f_{V_k}(v;p_k) = \delta(v-\gamma);$$

2) if $\alpha \neq 0$ and $\beta \geq 0$ or $\beta \leq -1$

$$f_{V_k}(v;p_k) = \frac{1}{2\sqrt{\alpha(v-\gamma)}}, \quad v_{\mathrm{b,min}} \leq v \leq v_{\mathrm{b,max}};$$

3) if $\alpha \neq 0$ and $-1 < \beta < 0$

$$f_{V_k}(v;p_k) = \begin{cases} \frac{1}{2\sqrt{\alpha(v-\gamma)}}, & v_{\mathrm{b,min}} \leq v \leq v_{\mathrm{b,max}} \\ \frac{1}{\sqrt{\alpha(v-\gamma)}}, & \gamma \leq v < v_{\mathrm{b,min}} \end{cases}$$

where $v_{\mathrm{b,min}} = \min\{\alpha\beta^2+\gamma, \alpha(\beta+1)^2+\gamma\}$ and $v_{\mathrm{b,max}} = \max\{\alpha\beta^2+\gamma, \alpha(\beta+1)^2+\gamma\}$.

Finally, averaging over p_k, the density of the conditional variance of the interference due to user k is given by

$$f_{V_k}(v) = \frac{1}{N_s}\sum_{p_k=0}^{N_s-1} f_{V_k}(v;p_k). \tag{37}$$

Taking ψ_k and P_k into consideration, the variance of the kth interferer is

$$\Phi_k \triangleq E_{\psi_k}\left\{\frac{P_k}{P_1}V_k\cos^2\psi_k\right\} = \frac{P_k}{2P_1}V_k. \tag{38}$$

The corresponding density function is

$$f_{\Phi_k}(\phi) = \frac{2P_1}{P_k} f_{V_k}\left(\frac{2P_1}{P_k}\phi\right). \tag{39}$$

Since the relative delays are independent for different users, the random variables $\{\Phi_k\}$ are also independent. The density function of the total variance $\Phi = \sum_{k=2}^{K}\Phi_k$ is given by

$$f_\Phi(\phi) = f_{\Phi_2}(\phi) * f_{\Phi_3}(\phi) * \ldots * f_{\Phi_K}(\phi). \tag{40}$$

With the density of the variance of the MAI, we are able to evaluate the density function of the SINR $f_{\gamma_s}(\gamma)$ via the relation

$$\gamma_s = \frac{1}{\frac{N_0}{E_s} + \frac{2}{N_s^2}\phi}. \tag{41}$$

If we define the union bound for the convolutional code as a function of γ_s

$$g(\gamma_s) \triangleq \sum_{d=d_{\mathrm{free}}}^{\infty} \beta_d Q\left(\sqrt{2d\gamma_s}\right) \tag{42}$$

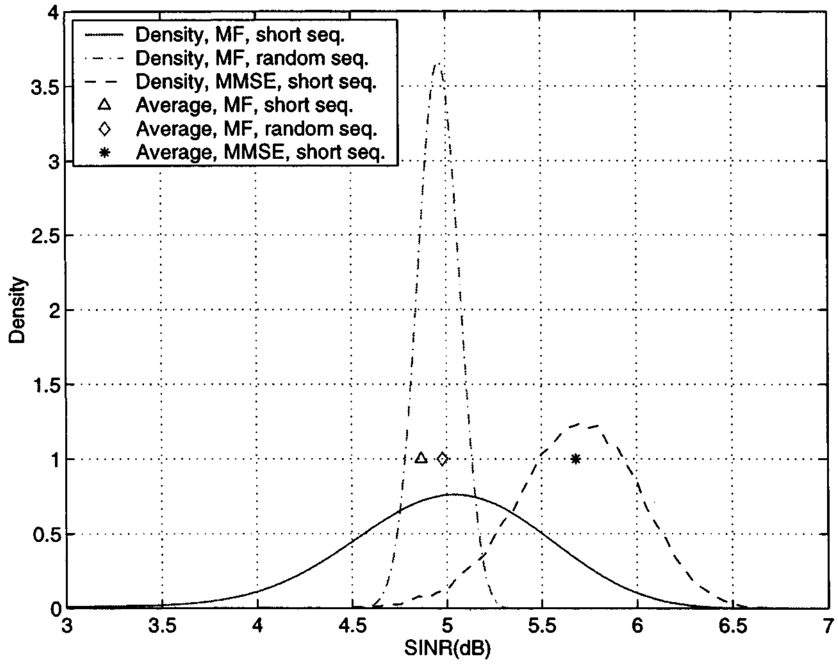

Fig. 2. Density functions of SINR for DS-CDMA system, $N_s = 31$ with Gold sequences, $K = 12$, $PR = 0$ dB, $R_c = 1/2$, $E_b/N_0 = 7$ dB.

the bit-error performance can be bounded by

$$P_e \leq \int_0^\infty g(\gamma) f_{\gamma_s}(\gamma) d\gamma. \tag{43}$$

This approach can also be applied to the system with random sequences. The variance of $W_{k,i}$ for any user k and any time i, conditioned on the delay τ_k, is given by [10] as

$$V_k \triangleq \mathrm{Var}\{W_{k,i}|\tau_k\} = \frac{N_s}{T_c^2}\left[\tau_k^2 + (T_c - \tau_k)^2\right]. \tag{44}$$

The random variable τ_k is uniformly distributed on $[0, T_c]$, leading to the distribution of V_k as

$$f_{V_k}(v) = \frac{1}{N_s\sqrt{\frac{2v}{N_s} - 1}}, \quad \frac{N_s}{2} \leq v \leq N_s. \tag{45}$$

Equations (39)–(43) can now be used to obtain the distribution of the SINR and the bounds on the bit-error probability for the long-sequence systems.

The density functions of the SINR for both short-sequence and long-sequence systems are shown in Fig. 2. Also shown are the average SINRs (i.e., GA) for both systems. Note that a similar figure is shown in [3], but it considers a multiple cell scenario and it is obtained by hybrid simulation/analysis, where the simulation is used to average over the random delays and the received power level. In Fig. 2, although the difference between the average SINRs is small, the spread of the SINR for the short-sequence system is significantly larger than that of the long-sequence system. The large spread is due to the probability that a user might be trapped in a disadvantageous delay setting with time invariant cross-correlations, and this leads to the performance degradation of the short-sequence system. We also show the density function of SINR for the short-sequence systems with the MMSE receiver to be described in Section IV, obtained by hybrid analysis/simulation. The MMSE receiver provides a significant improvement of the average SINR, and almost all of the density lies above the average MF receiver SINR.

In Fig. 3, several analytical BER estimates for the convolutionally-coded CDMA systems using short sequences are compared to computer simulation results. The processing gain N_s is 31, and there are 12 users in the system. Gold sequences are used in the short-sequence systems, with a rate-1/2, 32-state convolutional code. The results from the density function approach (labeled "Analysis" in Figs. 3 and 4) and the IGA are quite close to each other, and simulation results show they are accurate below a bit-error probability of 10^{-3}. The deviation at higher BER is due to the weakness of the union bound at low E_b/N_0 values. The GA results are also shown and are clearly too optimistic.

In Fig. 4, the results of using different analytical methods are shown for a convolutionally-coded system using long spreading sequences. Note that for the density function approach, a fixed sequence is assumed for the desired user. Obviously, the difference between the density function approach and the IGA is quite small. The simplest method, GA, is fairly accurate for a BER higher than 10^{-5}. Comparing to results in Fig. 3, it can be seen that the performance of the short-sequence system is worse than that of the long-sequence system, especially when there is a near–far problem (e.g., $P_k/P_1 = 3$ dB).

D. IGA on a Fading Channel

The IGA method described above can be easily extended to a frequency-nonselective fading channel. Assuming the same signals described above are transmitted over a flat fading channel, the received signal is

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k}\alpha_k(t) a_k(t-\tau_k) b_k(t-\tau_k)\cos(\omega_c t + \psi_k) + n(t) \tag{46}$$

where $\alpha_k(t)$ is the amplitude of the fading experienced by the k-th user. We assume the fading is Rayleigh with $E\left[\alpha_k^2\right] = 1$.

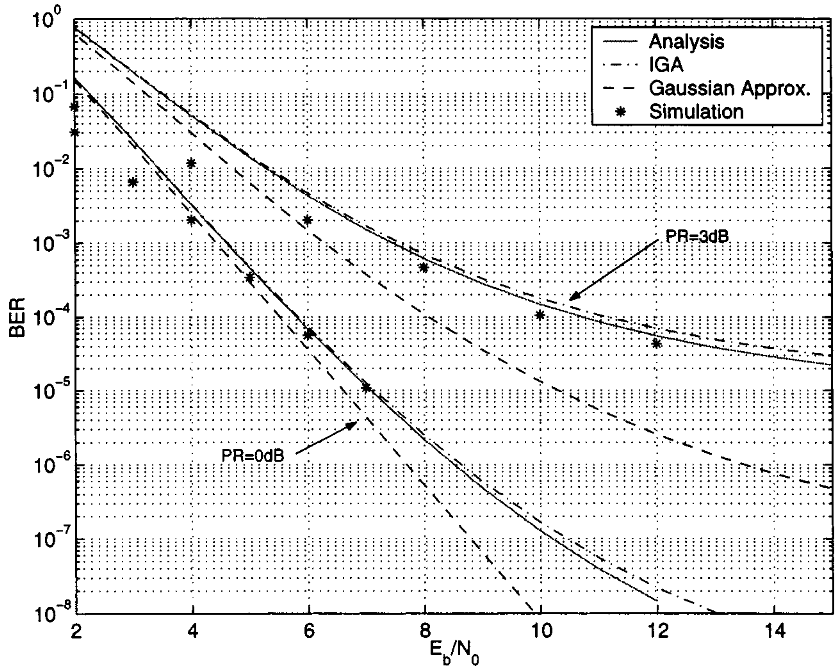

Fig. 3. Comparison of analytical methods for convolutionally-coded DS-CDMA system, AWGN channel, $N_s = 31$ with Gold sequences, $K = 12$, $R_c = 1/2$, 32-state code. $PR = P_k/P_1$ for all interfering users.

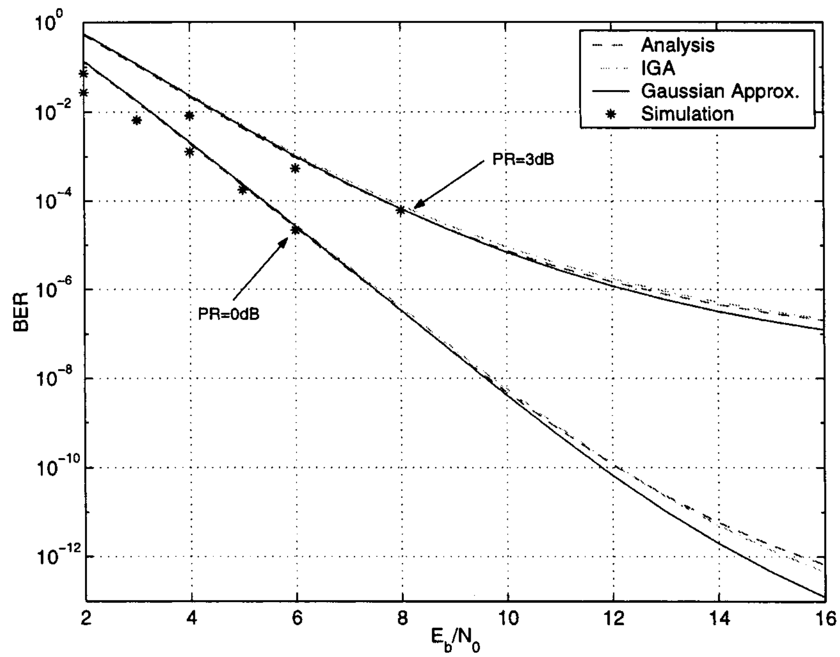

Fig. 4. Comparison of analytical methods for convolutionally-coded DS-CDMA system, AWGN channel, $N_s = 31$ with long sequences, $K = 12$, $R_c = 1/2$, 32-state code.

The normalized decision statistic for the i-th code symbol of the first user is

$$U_i' = \alpha_i^{(1)} N_s b_i^{(1)} + \sum_{k=2}^{K} \sqrt{\frac{P_k}{P_1}} \alpha_i^{(k)} I_k'(\mathbf{b}_{k,i}, \tau_k, \psi_{k,i}) + \eta_i'. \quad (47)$$

With the assumption of perfect interleaving, the $\left\{\alpha_i^{(k)}\right\}$ are independent for different time positions.

Given the delay τ_k, the conditional variance of the MAI is the same as that in the AWGN case, due to the normalization of the fading amplitude α_k. Following the steps from (37) to (41), the density of the SINR, $f_{\gamma_s}(\gamma)$, is obtained, as in the AWGN case. With the assumption of infinite interleaving and perfect channel state information, the union bound for the bit-error probability is given by (33), with the pairwise error probability given by [16]

$$P_2(d) = \left(\frac{1-\mu}{2}\right)^d \sum_{j=0}^{d-1} \binom{d-1+j}{j} \left(\frac{1+\mu}{2}\right)^j \quad (48)$$

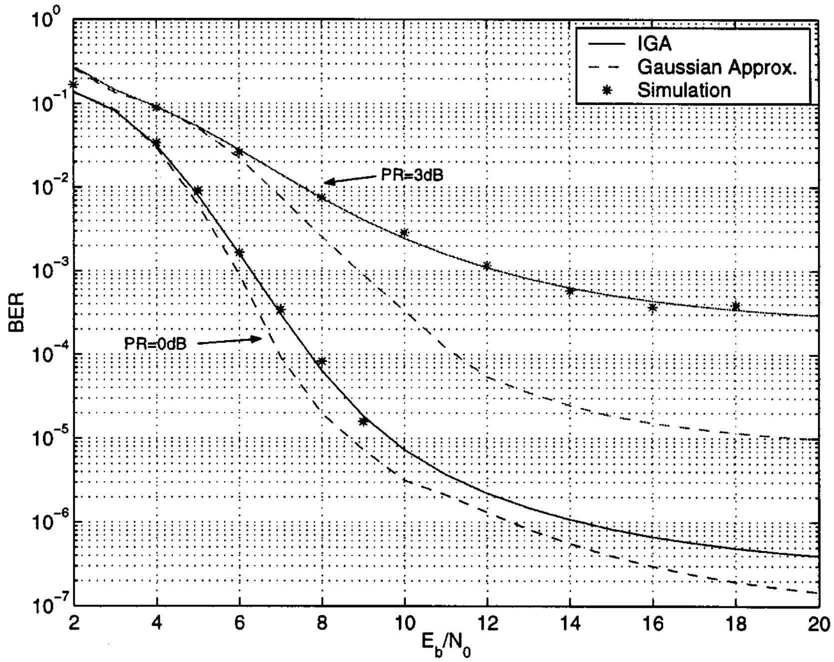

Fig. 5. Comparison of analytical methods for the turbo-coded DS-CDMA system, flat Rayleigh fading, $N_s = 31$ with Gold sequences, $K = 12$, $R_c = 1/2$, with different P_k/P_1 ratios.

where

$$\mu = \sqrt{\frac{\gamma_s}{1+\gamma_s}}. \tag{49}$$

Denoting the union bound for the Rayleigh fading channel as a function of γ_s by $h(\gamma_s)$, the final bit-error probability estimate is

$$P_e \leq \int_0^\infty h(\gamma) f_{\gamma_s}(\gamma) d\gamma. \tag{50}$$

The IGA can also be used to obtain the performance of a turbo-coded CDMA system. The turbo coding scheme considered here is the same as described in [17]. The turbo encoder consists of two identical rate-1/2 recursive systematic convolutional encoders, with the generating matrix $(1, (1+D^2)/(1+D+D^2))$. The two parity bit streams are punctured alternately, giving a code rate of 1/2. At the end of each transmission block, only the first encoder is driven back to the all-zero state through the transmission of tail-bits, whereas the other encoder is not terminated. For the first encoder, the trellis termination scheme of [18] is used, wherein the encoder feedback bit is taken as the encoder input and transmitted together with the parity bits. The two constituent encoders are concatenated in parallel through a pseudo-random permutation, with a block size of 190 information bits. For better performance, we adopted the so-called "S–Random" interleaver [18], which prohibits the mapping of a bit position to another within a distance $\pm S$ of a bit position already chosen in any of the S previous selections. For the block size of 190 bits, we chose such an interleaver with $S = 8$. The Log-MAP algorithm is used in the iterative decoder, and 15 iterations are used to assure convergence.

The performance of the turbo code is obtained analytically by the union bound, assuming a maximum-likelihood decoder is used. Most terms of the distance spectrum of the code are obtained based on the concept of "uniform interleaver," which represents an average of all possible interleaving permutations of the turbo code. In [17], the minimum distance is found to be 8 for the specific "S-Random" permutation we have chosen, rather than 3 for the "uniform interleaver," and thus the first few terms of the distance spectrum are modified accordingly to improve the accuracy of the union bound for BER $< 2 \times 10^{-5}$. To improve the accuracy for a BER higher than 2×10^{-5}, the $h(\gamma_s)$ function is implemented by interpolation from the simulation results. In Fig. 5, the IGA estimates for the turbo-coded CDMA systems using Gold sequences are compared to computer simulation results. The IGA results are quite close to the simulation results, for both the perfect power control case and when there is a near–far situation ($P_k/P_1 = 3$ dB). The GA results are also shown but again are too optimistic.

IV. MMSE Receiver

Since the relative delays change slowly compared to the data rate, the short sequence systems may be trapped in disadvantageous situations where the cross-correlations remain unchanged. On the other hand, the cyclostationary nature of the MAI for the short-sequence systems allows the application of an adaptive MMSE receiver to suppress the interference. The receiver uses a chip-matched filter followed by an adaptive equalizer structure that performs despreading and interference suppression [1], [2]. The number of taps of the adaptive filter is equal to N_s. In the following, we compare the performance of the short-sequence systems with both the MF receiver and the MMSE receiver to that of the long-sequence systems with the MF receiver. The analytical result for the MMSE receiver is obtained following the methods given in [2]. Given the delays of all interfering users, the optimum tap weights of the MMSE receiver are obtained by solving the Wiener–Hopf equation, and

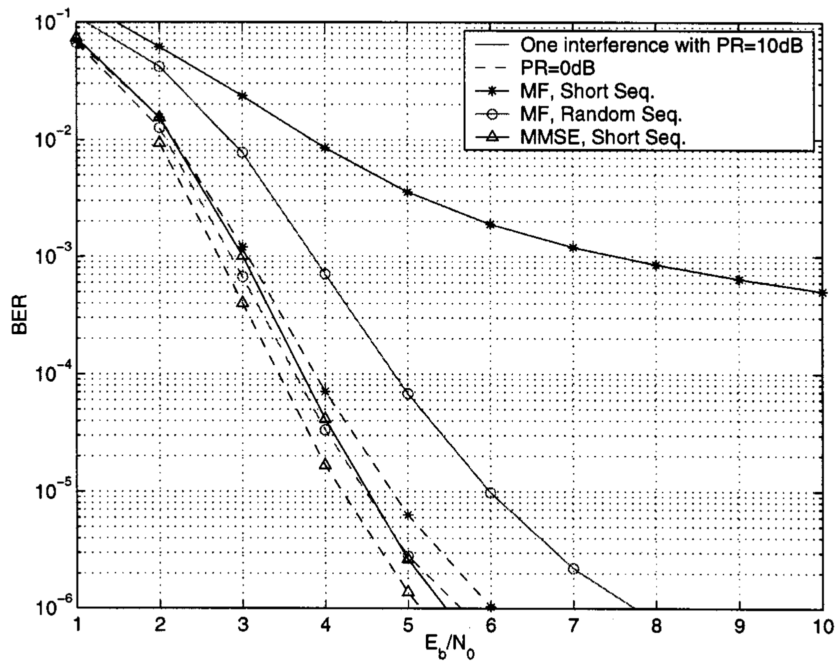

Fig. 6. Analytical results for turbo-coded DS-CDMA systems, AWGN, $N_s = 31, K = 12, R_c = 1/2$.

the conditional bit-error probability is estimated analytically. The final bit-error probability is obtained by taking a sample average for the various realizations of the delays. It was shown in [2], [17] that when perfect channel state information is available, practical adaptive algorithms (e.g., RLS) can provide performance quite close to that of optimum MMSE tap weights.

In addition to the case of perfect power control, we also consider a near–far situation when there is only one strong interferer, while other users keep the same power level as the desired user. This scenario attempts to model a cell containing mobiles with perfect power control with one uncontrollable interfering user from the adjacent cells.

Fig. 6 shows the analytical estimates of the performance of the turbo-coded CDMA systems on the AWGN channel. When perfect power control is maintained, the system can achieve a BER of 10^{-4} with the E_{b}/N_0 below 4 dB. In this case, the thermal noise is the dominating factor for errors, and the difference between the short and long-sequence systems is small. The advantage of the MMSE receiver is also marginal. However, when there is at least one strong interfering user, the difference between the short and long-sequence systems becomes obvious since a bad delay of the strong interferer may impair the performance of the desired user of the short-sequence system. On the other hand, since the strong interference is confined to a subspace with a small number of dimensions, the MMSE receiver can suppress it effectively and provides excellent performance. Compared with the case of perfect power control, we find that the power penalty of one strong interferer is smaller than 0.3 dB when the MMSE receiver is used.

Fig. 7 shows the analytical estimates of the performance of the turbo-coded CDMA systems on a flat Rayleigh fading channel. It is assumed that infinite interleaving results in independent fades on each bit, and perfect knowledge of the channel state information (CSI) is available to the receiver. For the MMSE receiver, it is assumed that the adaptive algorithm is not able to track the fades on any of the interfering users. As a consequence, when the optimum tap weights are derived, the expectation of the square error is carried out over the random fades [2]. Referring to Fig. 6, it can be seen that, for the case of perfect power control, the systems require higher values of E_{b}/N_0 on the fading channel. Also, the difference between the short-sequence and long-sequence systems is larger, and the MMSE receiver provides moderate improvement. When there is one strong interfering user, the long-sequence system requires $E_{\mathrm{b}}/N_0 = 8.2$ dB to achieve a BER of 10^{-3}, while the short-sequence system with the MF receiver appears to bottom out at a BER well above 10^{-3}. However, with the MMSE receiver, the short-sequence system significantly outperforms the long-sequence system. Compared to the case of perfect power control, the MMSE receiver achieves a BER of 10^{-4} with a penalty of only 0.4 dB.

Finally, the performance of the turbo-coded CDMA systems over a frequency-selective Rayleigh fading channel, obtained by computer simulation, is shown in Fig. 8. The number of resolvable paths is chosen to be $L_p = 3$, with a constant multipath intensity profile (MIP). The conventional RAKE receiver with L_p branches is used, with either a standard MF or an MMSE filter on each branch of the RAKE. The MMSE filter coefficients for each tap of the RAKE are determined by independently minimizing the mean-square errors of each tap [2]. It is also assumed that the fades on the resolvable paths of the channel are independent of each other, and the fades of any interfering users are too fast for the adaptive algorithm to track.

Comparing to Fig. 7 for the case of perfect power control, we find that the additional diversity provided by the multipath channel allows the system to operate at lower values of E_{b}/N_0. For these parameter values, it can be shown that the thermal noise is the dominating factor. Thus, the difference between the short-sequence and long sequence systems is minimal, and the advantage of the MMSE receiver over the MF receiver is mar-

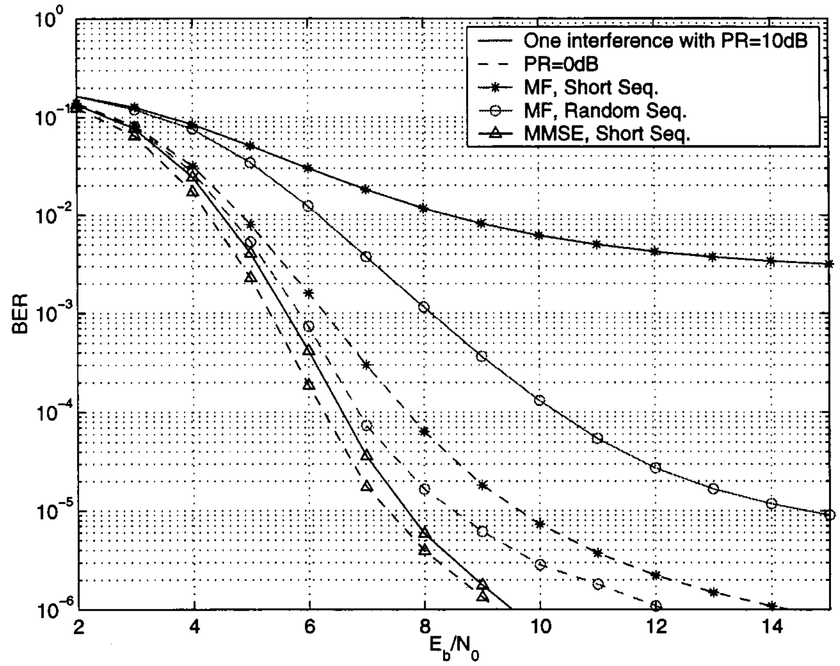

Fig. 7. Analytical results for turbo-coded DS-CDMA systems, flat Rayleigh fading, $N_s = 31$, $K = 12$, $R_c = 1/2$.

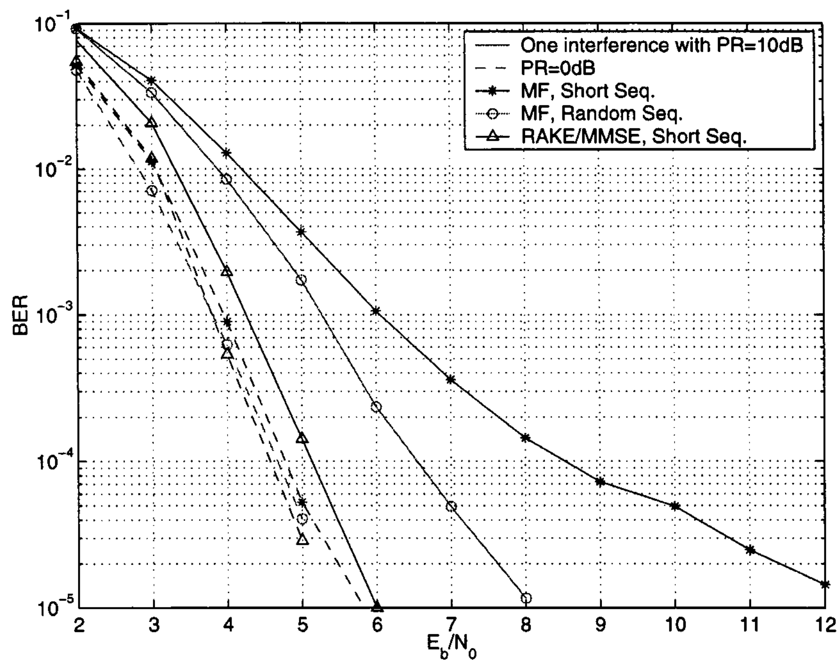

Fig. 8. Simulation results for turbo-coded DS-CDMA systems, frequency-selective Rayleigh fading, $L_p = 3$, constant MIP, $N_s = 31$, $K = 12$, $R_c = 1/2$.

ginal. When there is a near–far problem, the performance of the MF receiver is degraded relative to the case of perfect power control but is significantly better than that found in flat fading. We also note that the difference between the short-sequence and long-sequence systems is smaller than that on either the AWGN channel or the flat fading channel. The reason appears to be that the power of the interference is now spread over L_p paths, and the cross-correlations are averaged over several relative delays, reducing the effect of the desired signal being trapped in an inferior setting. On frequency-selective fading channels, the improvement in performance provided by the MMSE/RAKE receiver is generally less dramatic than that on the flat fading channel. As explained in [2], when the RAKE/MMSE is not able to track the fading of the interfering users on the frequency-selective fading channel, the dimensionality of the interference subspace is typically $2(K-1)L_p$, too large for the MMSE receiver to effectively suppress the interference. However, as shown in Fig. 7, the RAKE/MMSE receiver still provides significant improvement when there is only one strong interfering user, since the dimensionality of the strong interference is small, even though it has been multiplied by the number of paths of the frequency-selective channel.

V. Conclusion

Several analytical methods were compared for the performance evaluation of coded CDMA systems with MF receiver. While the standard GA is fairly accurate for long-sequence systems, it gives optimistic estimates for short-sequence systems. It is shown that the IGA produces results almost as accurate as those of the density function approach but with much less complexity. The performance differences between long sequence and short sequence spreading in a turbo-coded CDMA system were investigated for different channels and different near–far situations, and the conclusions agree with those previously found for uncoded or convolutionally-coded CDMA systems. With the MF receiver, there is a degradation in the average performance of the short sequences compared to that of the long sequences. However, the MMSE receiver for the short sequence systems provides the best average performance on all the channels considered, especially when there is a near–far problem.

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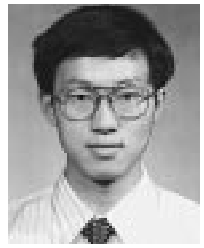

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