On Viterbi Detector Path Metric Differences

Andrei Vityaev and Paul H. Siegel, *Fellow, IEEE*

***Abstract*— This letter continues the investigation of methods for computing exact bounds on the path metric differences in maximum-likelihood sequence detectors based upon the Viterbi algorithm. New upper and lower estimates for these bounds are presented and recast in terms of a collection of linear programming problems. These estimates improve upon previously proposed linear programming bounds. The estimates are applied to derive exact bounds or provably close to exact bounds for several Viterbi detectors corresponding to coded and uncoded partial-response channels of practical interest in digital magnetic and optical recording.**

***Index Terms*— Magnetic recording, partial response channels, path metric differences, Viterbi algorithm.**

I. Introduction

THE VITERBI algorithm (VA) is widely used in digital communications and recording to implement maximum-likelihood (ML) sequence estimation of signals generated by an underlying Markov chain and corrupted by additive white Gaussian noise (AWGN). An important problem in efficient circuit implementation of the ML detector is the determination of the observable range of the path metric differences. Tight bounds on the extremes of these differences are valuable for a number of reasons. They bear upon requirements for the dynamic range of the arithmetic section of the add-compare-select (ACS) processor, the circuit data path width, and the implementation of path metric normalization techniques, particularly the modular renormalization approach based upon two's complement number representations and arithmetic [4], [8].

In [1], the problem of finding exact difference metric bounds was addressed in the context of uncoded and coded binary-input partial-response channels arising in digital recording applications. A method of computing upper estimates of the bounds by solving a collection of linear programming (LP) problems was developed. This method, referred to as the LP bound, was applied in [1] to determine the exact bounds for some simple examples of interest in digital recording: the binary-input, dicode partial-response channel, the biphase-coded dicode partial-response channel, and the binary-input, class-2 partial-response (PR2) channel for magnetic recording. Upper estimates were also calculated for the exact bounds of the even-mark-modulation (EMM) coded class-1 (duobinary or PR1) partial-response channel for optical recording. For systems described by trellis structures with more than four states, the estimates are much more difficult to apply and, if applied, are not guaranteed to be tight.

A more challenging problem, addressed in [2], is the complete characterization of the state-space of the path metric differences, as well as the determination of the steady-state probability distribution of these differences as a function of the additive noise statistics. By examining the one-step dynamics of the binary-input dicode channel and the EMM-coded duobinary channel, the recurrent region of the state-space containing the all-zero path metric difference state was calculated. For the dicode channel, a closed-form expression was derived for the steady-state distribution as a function of the noise statistics. For the EMM-coded duobinary channel, the state-space characterization verified the tightness of a conjectured set of exact bounds that the LP methods were not able to confirm. More recently, this approach has been successfully applied to the characterization of the recurrent region for the class-2 partial-response channel, but the analysis is extremely intricate [3]. For channels with more than four states, the analytic state-space characterization becomes infeasible.

In this letter, we make several contributions to the theory and application of methods for computing exact bounds on path metric differences. In Section II, we introduce a sequence of computable upper estimates that can improve upon the estimates obtained using the methods mentioned above. We also derive a sequence of lower estimates. In Section III, we give a partial characterization of the received vectors that achieve the LP bounds in [1]. In Section IV, we formulate the upper estimates and lower estimates in terms of a collection of LP problems. Section V gives some applications of the new estimates. We calculate improved, and in some cases tight, estimates of the survivor path metric difference bounds for a number of systems of practical interest that could not be easily treated using the earlier approaches, including EMM-code partial-response class-2 (PR2); $d = 1$ constrained extended partial-response class-4 (EPR4); $d = 1$ constrained, doubly-extended partial-response class-4 (E^2PR4).

Paper approved by T. Aulin, the Editor for Coding and Communications Theory of the IEEE Communications Society. Manuscript received August 20, 1997; revised February 5, 1998 and May 18, 1998. This work was supported in part by the National Science Foundation under Grant NCR-9612802. This paper was presented in part at the 1996 IEEE Global Telecommunications Conference (GLOBECOM'96), London, U.K., November 1996.

A. Vityaev was with the Department of Mathematics, University of Washington, Seattle, WA 98195. He is now with Marvell Semiconductor, Inc., Sunnyvale, CA 94086 USA (e-mail: avityaev@marvell.com).

P. H. Siegel is with the University of California at San Diego, La Jolla, CA 92093-0407 USA (e-mail: psiegel@ucsd.edu).

Publisher Item Identifier S 0090-6778(98)09400-8.

II. Upper and Lower Estimates for the Bounds

Given a state s in the detector trellis T, let $P_k(s)$ denote the depth-k predecessor set of s, meaning the set of states in T from which s can be reached by a path of length k edges. For a state u in $P_k(s)$, the set of output sequences generated by paths from u to s of length k is denoted $E_k(u, s)$, and the set of all paths of length k ending at state s, that is $\cup_{u\in P_k(s)} E_k(u, s)$, is denoted by $E_k(s)$. For a pair of states s and t, we let $c(s, t)$

denote the minimum k such that $P_k(s) = P_k(t)$, and denote the common predecessor set by $P_{c(s,t)}$.

For any given path e we denote the edges of e by $e_1, \cdots, e_k$ and we denote the states through which e passes by $e(0), \cdots, e(k)$. We will also use the notation $e_1, \cdots, e_k$ for the symbols generated by the corresponding edges. The meaning will be clear from context. For any sequence of measured signals r we denote by $f(r, e)$ the linearized path metric

$$f(r, e) = \sum_{i=1}^{k} e_i^2 - 2r_i e_i.$$

In this letter, we will assume that the sequence of measured signals $r = (r_1, \cdots, r_k)$ satisfies $r_j \in [-R, R]$ for every j. Suppose that the path e is the survivor path of length k with $e(k) = s$. Let $m_k(s) = f(r, e)$ be the corresponding survivor path metric. Denote the survivor path metric difference of states s and t at time k by

$$DM_k(s, t) = m_k(s) - m_k(t).$$

The following estimate was announced in [1].

A. Proposition

Let $\delta = c(s, t)$. At any time $k \geq \delta$ the difference metric at time k, $DM_k(s, t)$, satisfies

$$-\max_u \Delta_u(t, s) \leq DM_k(s, t) \leq \max_u \Delta_u(s, t)$$

where

$$\Delta_u(s, t) = \max_{r \in [-R, R]^\delta} \left(\min_{e \in E_\delta(u, s)} f(r, e) - \min_{e' \in E_\delta(u, t)} f(r, e') \right). \quad (1)$$

Proof: The inequalities follow immediately from the analysis of the VA dynamics on the depth-δ butterflies connecting states $u, v \in P_\delta$ and states s, t. □

We will now develop a sequence of upper and lower estimates for the exact upper bound on $DM_k(s, t)$. The estimates are applicable to any finite state trellis structure, and represent an improvement with respect to the estimates in [1]. First, we state an elementary lemma that will prove to be useful in the derivation and analysis of the upper and lower estimates.

B. Lemma

The estimate $\Delta_u(s, t)$ can be rewritten in the following form:

$$\begin{aligned}\Delta_u(s, t) &= \max_{r \in [-R, R]^\delta} \max_{e' \in E_\delta(u, t)} \min_{e \in E_\delta(u, s)} (f(r, e) - f(r, e')) \\ &= \max_{e' \in E_\delta(u, t)} \max_{r \in [-R, R]^\delta} \min_{e \in E_\delta(u, s)} (f(r, e) - f(r, e')).\end{aligned}$$

Proof: It is clear that the first equation gives the same result as (1). The order of the maximization operations can then be interchanged without affecting the result, thereby giving the second equation. □

The new upper estimates for the exact bounds are given by the following proposition.

C. Proposition

Suppose that $c(s, t) = \delta$. Then for any $k \geq \delta$, any $k' \geq k$, the following inequality holds:

$$-\Delta^k(t, s) \leq DM_{k'}(s, t) \leq \Delta^k(s, t) \quad (2)$$

where

$$\Delta^k(s, t) = \max_{e' \in E_k(t)} \max_{r \in [-R, R]^k} \min_{e \in E_k(e'(0), s)} (f(r, e) - f(r, e')). \quad (3)$$

Proof: The maximum possible value of the metric difference $DM_{k'}(s, t)$, viewed as a function of the received vector $r \in [-R, R]^{k'}$, is given by

$$\max_{r \in [-R, R]^{k'}} DM_{k'}(s, t) = \max_{r \in [-R, R]^{k'}} \max_{e' \in E_{k'}(t)} \min_{e \in E_{k'}(s)} (f(r, e) - f(r, e')). \quad (4)$$

By Lemma II-B, the maximum value of $DM_{k'}(s, t)$ can be rewritten as

$$\max_{e' \in E_{k'}(t)} \max_{r \in [-R, R]^{k'}} \min_{e \in E_{k'}(s)} (f(r, e) - f(r, e'). \quad (5)$$

We can consider shorter paths (of length k, rather than k') and rewrite (5) as

$$\max_{e' \in E_k(t)} \max_{r \in [-R, R]^k} \min_{e \in E_k(s)} ((f(r, e) + m(e)) - (f(r, e') + m(e'))) \quad (6)$$

where $m(e)$ and $m(e')$ are the metrics of the survivor paths preceding e and e', respectively. Note that the elements of sets $E_k(s)$ and $E_k(t)$ are subsequences of the sequences which comprise $E_{k'}(s)$ and $E_{k'}(t)$, respectively. By taking $\min_e$ in (6) over a smaller set $E_k(e'(0), s) \subset E_k(s)$ we can only increase (6), and thus

$$DM_{k'}(s, t) \leq \max_{e' \in E_k(t)} \max_{r \in [-R, R]^k} \min_{e \in E_k(e'(0), s)} (f(r, e) + m(e)) - (f(r, e') + m(e')). \quad (7)$$

Since now e and e' start from the same state, we have $m(e) = m(e')$. Therefore, the right side of (7) is equal to $\Delta^k(s, t)$. □

1) Remark: This proof of the upper estimates provides an alternative proof to Proposition II-A.

We now state the lower estimates for the exact upper bound.

D. Proposition

Suppose b is the exact upper bound for $DM(s, t)$. Then the following inequality holds for any k.

$$b \geq \max_{e' \in E_k(t)} \max_{r \in [-R, R]^k} \min_{e \in E_k(s)} (f(r, e) - f(r, e')). \tag{8}$$

Proof: The maximum possible value of the metric difference $DM(s, t)$ is given by

$$\max_k DM_k(s, t) = \max_k \max_{r \in [-R, R]^k} \max_{e' \in E_k(t)} \min_{e \in E_k(s)} (f(r, e) - f(r, e')).$$

The proposition follows from an application of Lemma II-B. □

2) Remark: Note that the only difference between the upper estimate (3) and the lower estimate (8) is that we impose the restriction that e and e' start from the same state. We conjecture that the upper and lower estimates converge to the exact upper bound on the survivor path metric difference.

III. Characterization of External Points

For trellis structures having the property that for $u \in P_\delta$, there is a unique path sequence $e \in E_\delta(u, s)$ and a unique path in $e' \in E_\delta(u, t)$, the estimate $\Delta_u(s, t)$ is achieved at a vertex r of the region $[-R, R]^\delta$. This class of trellises includes those based upon deBruijn graphs which arise in connection with binary-input partial-response systems. The following result extends this characterization of the optimal vector $r \in [-R, R]^\delta$ to trellis structures, such as that occurring in the EMM-coded duobinary (PR1) case, where $|E_\delta(u, s)| = n > 1$.

A. Proposition

Suppose there exist n paths from u to s. Then we have

$$\Delta_u(s, t) = \Delta_u(s, t)|_{r=r^*},$$

where at least $\delta - n + 1$ coordinates of r^* have the absolute value equal to R. That is, up to $n-1$ coordinates may have absolute value strictly less than R.

Proof: Recall from Lemma II-B that

$$\Delta_u(s, t) = \max_r \max_{e'} \min_e (f(r, e) - f(r, e')) = \max_{e'} \max_r \min_e (f(r, e) - f(r, e')).$$

Thus it is enough to prove the result for a fixed path e'. Let $e^1, \cdots, e^n$ be all the paths from u to s. Note that $L_j(r) = f(r, e^j) - f(r, e')$ are linear functions of r. Therefore, we have to study the maximum of the minimum of linear functions:

$$\max_r \min\{L_1(r), \cdots, L_n(r)\} = \max_r F(r). \tag{9}$$

Suppose the maximum occurs at a point r^* where

$$|r_1^*| \neq R, \cdots, |r_n^*| \neq R.$$

We would like to prove that in this case there exist $\tilde{r}_1, \cdots, \tilde{r}_n$ such that

$$F(\tilde{r}_1, \cdots, \tilde{r}_n, r_{n+1}^*, \cdots, r_\delta^*) = F(r^*) \tag{10}$$

and such that $|\tilde{r}_j| = R$ for some $1 \leq j \leq n$.

We start proving it with the following claim. Let us denote $\alpha_1 = L_1(r^*), \cdots, \alpha_n = L_n(r^*)$.

Claim: The following system of n linear equations

$$\begin{aligned} L_1(r_1, \cdots, r_n, r_{n+1}^*, \cdots, r_\delta^*) &= \alpha_1 \\ &\vdots \\ L_k(r_1, \cdots, r_n, r_{n+1}^*, \cdots, r_\delta^*) &= \alpha_n, \end{aligned} \tag{11}$$

and of n unknowns $(r_1, \cdots, r_n)$ has at least two solutions.

Proof of the claim: Since each of the L_j's is a linear function, it can be written as

$$L_j(r_1, \cdots, r_n) = L_{j0} + L_{j1}r_1 + L_{j2}r_2 + \cdots + L_{jn}r_n$$

where we suppressed $r_{n+1}^*, \cdots, r_\delta^*$ in the notations. Let $\mathcal{L}$ be the matrix comprised of coefficients L_{ji}, $i \neq 0$:

$$\mathcal{L} = \begin{pmatrix} L_{11} & \cdots & L_{1n} \\ \cdots & \cdots & \cdots \\ L_{n1} & \cdots & L_{nn} \end{pmatrix}.$$

We know that $(r_1^*, \cdots, r_n^*)$ is a solution of the system (11). Suppose that it is the only solution. In that case, matrix $\mathcal{L}$ is invertible. Let v be the n-vector given by

$$v = \mathcal{L}^{-1}(1, 1, \cdots, 1)'.$$

The point $(r_1^*, \cdots, r_n^*)$ is an interior point of the hypercube $[-R, R]^n$. Therefore, the point

$$(r_1^* + \epsilon v_1, \cdots, r_n^* + \epsilon v_n) \tag{12}$$

is an interior point of the same hypercube for $\epsilon > 0$ small enough.

If we evaluate functions L_j at the point (12) then we find that

$$L_j(r_1^* + \epsilon v_1, \cdots, r_n^* + \epsilon v_n) = L_j(r_1^*, \cdots, r_n^*) + \epsilon$$

for every j. This contradicts the assumption that the maximum of (9) is achieved at $(r_1^*, \cdots, r_n^*, \cdots, r_\delta^*)$. This contradiction establishes the claim. □

Proof of the Proposition (continued): The claim implies that there exists a line Λ such that

$$L_j(r_1, \cdots, r_n, r_{n+1}^*, \cdots, r_\delta^*) = \alpha_j \; \forall j \text{ for } (r_1, \cdots, r_n) \in \Lambda.$$

Let $(\tilde{r}_1, \cdots, \tilde{r}_n)$ be the point of intersection of Λ with the boundary of the n-dimensional hypercube $[-R, R]^n$. Then (10) holds, which proves the proposition. □

1) Example: In [1], it was pointed out that the maximum value of the difference metric $\Delta_1(1, 3)$ for the EMM-coded duobinary (PR1) channel does not occur at a vertex of the sample space $[-2, 2]^3$. In fact, it follows from the characterization in [2] of the state-space of the difference metrics that the exact bound $DM(1, 3) = 9$ is achieved by $\Delta_3(1, 3)$ at the point $r = (-2, 0, 2)$.

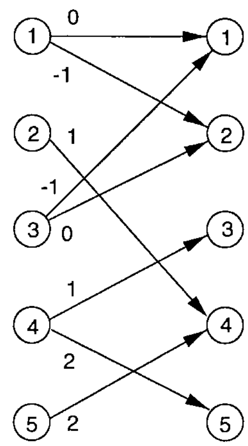

Fig. 1. Trellis for EMM-coded partial response class-2 (PR2).

2) Example: The EMM-coded partial-response class-2 (PR2) channel [5] is described by a 5-state trellis shown in Fig. 1. There are two paths of length 4 from state 3 to 4, corresponding to the state sequences $3, 1, 1, 2, 4$ and $3, 2, 4, 5, 4$. These generate the output sequences $(-1, -2, -1, 1)$ and $(0, 1, 2, 2)$, respectively. There is only one path from state 3 to 5, with state sequence $3, 1, 2, 4, 5$, generating the output sequence $(-1, -1, 1, 2)$. If the sample range is taken to be $[-4, 4]$, the maximum value of $\Delta_3(4, 5)$, namely $50/3$, is achieved by the length-4 sample vector $(-4, -11/3, 4, 4)$.

IV. Reduction to Linear Programming

We now show how the upper and lower estimates (3) and (8) can be computed using linear programming [7].

Linear Programming for Upper Estimate:

1) For all possible paths $e' \in E_k(t)$, solve:
Maximize $\tau = \tau(e')$ subject to:
There exists r such that

$$\begin{aligned} f(r, e^1) - f(r, e') &\geq \tau \\ &\vdots \\ f(r, e^n) - f(r, e') &\geq \tau \\ r_j &\leq R, \text{ for all } j \\ r_j &\geq -R, \text{ for all } j. \end{aligned}$$

2) Here $e^1, \cdots, e^n$ are all possible paths from $e'(0)$ to s. The upper estimate is equal to $\max_{e'} \tau(e')$.

Remark: This is a collection of LP problems in $k+1$ dimensional space (r, τ) with $n + 2k$ linear constraints. Finding the lower estimate (8) can be reduced to the problem above by removing the restriction on e. It is easy to see that the complexity of the collection of LP problems grows exponentially as k increases. On the other hand, software packages are available for solving LP problems with up to hundreds of thousands of variables. For trellises with no more than 10 states, the estimates are generally sufficient to determine the minimum number of bits required to span the range of path metric differences.

TABLE I
Upper and Lower Estimates for Exact Upper Bounds: Short Path Lengths

	DM(1,2)	DM(3,2)	DM(4,2)	DM(5,2)
upper estimate k=4	11	54	31	65
lower estimate k=7	11	49	27	60

TABLE II
Upper and Lower Estimates for Exact Upper Bounds: Long Path Lengths

	DM(1,2)	DM(3,2)	DM(4,2)	DM(5,2)
upper estimate, k=12	-	50	27	61
lower estimate, k=13	-	49.4	27	60.4

TABLE III
Upper Lower Estimates for Exact Lower Bounds: Short Path Lengths

	DM(2,1)	DM(2,3)	DM(2,4)	DM(2,5)
upper estimate, k=4	7	42	27	47
lower estimate, k=7	7	32.75	25.92	37.75

TABLE IV
Upper and Lower Estimates for Exact Lower Bounds: Long Path Lengths

	DM(2,1)	DM(2,3)	DM(2,4)	DM(2,5)
upper estimate, $k = 10$	-	34	26.3333	39
lower estimate, $k = 2$	-	33.3333	26.1111	38.3333

V. Applications

We choose three examples to illustrate the improved linear programming methods for computing estimates of the exact bounds.

A. Bounds for EMM Partial Response Class-2 (PR2)

The first example is the EMM-constrained partial response class-2 (PR2) optical recording channel, with trellis shown in Fig. 1.

Let $[-4, 4]$ be the admissible range for the noisy, received samples. Using the sequence of upper and lower estimates, we can find bounds for difference metrics $DM(1, 2)$, $DM(3, 2)$, $DM(4, 2)$, $DM(5, 2)$ which are within 2% from the exact upper and lower bounds.

First we compute upper and lower estimates for the upper bound for moderately short lengths of paths $k = 4$ and $k = 7$ (see Table I).

We see that while we have an optimal upper bound for $DM(1, 2)$, other metrics require longer paths. Table II gives the tighter upper and lower estimates that are within 2% from the exact.

Now, we turn to the lower bounds. To find them, we find the upper bounds for $DM(2, 1)$, $DM(2, 3)$, $DM(2, 4)$, $DM(2, 5)$ (see Tables III and IV).

Thus, we conclude that

$$\begin{aligned} -7 \leq DM(1, 2) \leq 11, \qquad & -34 \leq DM(3, 2) \leq 50 \\ -26.3333 \leq DM(4, 2) \leq 27, \qquad & -39 \leq DM(5, 2) \leq 61, \end{aligned}$$

where the bounds are within 2% from optimal.

In order to find the bound on the difference between the largest and the smallest metric difference, one has to compute

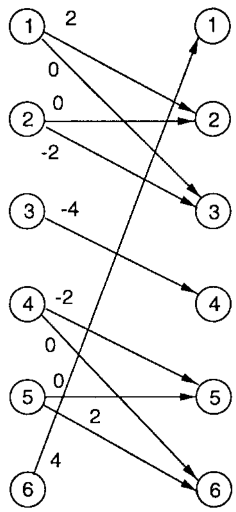

Fig. 2. Trellis for EPR4 with $d = 1$ constraint.

bounds for $DM(s, t)$ for all possible states s and t. It turns out that the largest difference metric is $DM(1, 5)$, whose maximum possible value is bracketed between the lower estimate 65.333 and the upper estimate 66. In other words, 66 is the bound within 2% from optimal. For comparison, the method in [4] gives 288 as a bound.

B. Bounds for $d = 1$ Constrained EPR4 Channel

The second example is the extended partial-response class-4 (EPR4) channel with the precoded $d = 1$ constraint [6]. The detector trellis is given in Fig. 2.

The correspondence between states and channel memory is given by: 1 = 011, 2 = 111, 3 = 110, 4 = 100, 5 = 000, 6 = 001.

We assume that the range of noisy, received samples is [-8, 8]. In the case of $d = 1$ constrained EPR4, it was possible to find the exact bound using the paths of length $k = 7$ for both upper and lower estimates.

The exact bounds for the metric differences are given by

$$
\begin{aligned}
-228 &\le DM(2, 1) \le 132\\
-256 &\le DM(3, 1) \le 168\\
-304 &\le DM(4, 1) \le 304\\
-276 &\le DM(5, 1) \le 84\\
-248 &\le DM(6, 1) \le 56.
\end{aligned}
$$

C. Bounds for $d = 1$ Constrained E^{2}PR4 Channel

The third example is the $d = 1$ constrained, doubly-extended partial-response class-4 (E^{2}PR4) channel. (See also [6] and [7].)

The detector trellis is given in Fig. 3 and the channel memory is given by: 1 = 0111, 2 = 1111, 3 = 1110, 4 = 1100, 5 = 1001, 6 = 0011, 7 = 0110, 8 = 1000, 9 = 0000, 10 = 0001.

We assume that the range of noisy, received samples is [–12, 12].

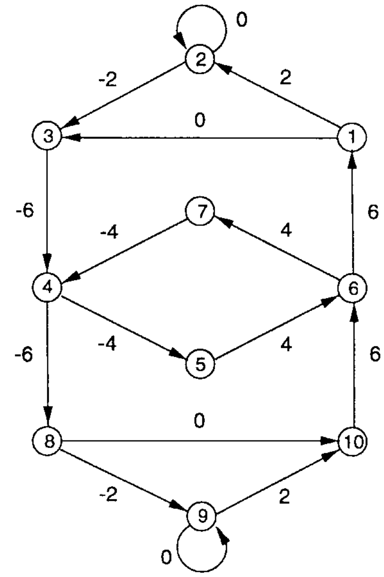

Fig. 3. Trellis for E^{2}PR4 channel with $d = 1$ constraint.

Here are the bounds for metric differences which are within 1% from optimal.

$$
\begin{aligned}
-668 &\le DM(2, 1) \le 456\\
-712 &\le DM(3, 1) \le 506\\
-820 &\le DM(4, 1) \le 676\\
-884 &\le DM(5, 1) \le 844\\
-716 &\le DM(6, 1) \le 148\\
-68 &\le DM(7, 1) \le 28\\
-912 &\le DM(8, 1) \le 912\\
-900 &\le DM(9, 1) \le 308\\
-856 &\le DM(10, 1) \le 264.
\end{aligned}
$$

Computing these bounds required estimates with the path length equal to 11.

VI. Computer Simulations

Given the results presented in the previous sections, a natural question arises whether the bounds given above can be found through computer simulation of the Viterbi detector. In this section, we describe the results of one such simulation. For the E^{2}PR4 channel with $d = 1$ constraint, the upper bound for $m(2) - m(1)$ was found to be 456. Moreover, the lower and upper estimates for this bound are only 1% apart, and thus 456 is an estimate with 1% accuracy. Besides finding the upper bound, the linear programming approach presented in this letter gives us a sequence of 13 received samples

$$
\begin{aligned}
&-10.5,\ 12,\ 12,\ -12,\ 7.09,\ 12,\ 10.23,\\
&-12,\ 8.59,\ 12,\ 12,\ -12,\ -12,
\end{aligned}
$$

which will drive the metric differences in the Viterbi detector from all zeros to $m(2) - m(1) = 454.36$.

The Viterbi detector was then simulated for the same channel. Received samples were taken to be uniformly distributed in the interval [–12, 12]. A uniform distribution of samples, rather than that which would be observed on the AWGN channel, was chosen in order to boost the frequency of occurrence of extreme values of the metric differences.

The Viterbi detector path metric differences were all initialized to zero. The results of the detector simulation are shown below. The left column contains the number of received samples passed through the Viterbi detector during the simulation and the right column contains the largest value of the metric difference $m(2) - m(1)$ observed during the simulation.

random received samples:	largest observed $m(2) - m(1)$
1×10^6	368.3
10×10^6	395.0
50×10^6	400.6
100×10^6	404.4

The results show that simulation with uniformly distributed received samples gives a bound 13% lower than the estimate upper bound, even after 100 million samples. When the more accurate AWGN model of the physical channel is used, simulations generally require even a larger number of samples to reveal comparably large difference metric values.

In practice, register overflows caused by underestimating the maximum magnitude of difference metrics may produce long bursts of detector errors that seriously degrade overall system performance. The empirical results confirm that, depending upon system error-rate specifications, there may be risks in substituting approximate bounds deduced from simulation for analytically derived bounds such as those presented in this letter.

VII. Conclusions

This letter presents an improved linear programming method for determining upper and lower estimates of the exact bounds on Viterbi detector path metric differences. These tighter estimates can be used to reduce the number of bits of precision needed to represent these metric differences in circuit implementations. The method reproduces the exact bounds found by earlier methods for the binary-input dicode and class-2 partial-response channels. In addition, it provides for the first time either exact bounds, or tight enough estimates to determine the minimum required number of bits of resolution, for several channels of practical interest in digital recording.

Acknowledgment

The authors would like to thank Philip E. Gill of the Department of Mathematics at the University of California at San Diego, for providing a copy of his linear programming software package and for his expert advice on numerical optimization.

References

[1] R. Behrens and A. Armstrong, "An advanced read/write channel for magnetic disk storage," in *Proc. 26th Asilomar Conf. Signals, Syst., Computers,* Pacific Grove, CA, Oct. 1992, pp. 956–960.

[2] A. R. Calderbank, P. Fishburn, and P. Siegel, "State-space characterization of Viterbi detector path metric differences," in *Proc. 1992 IEEE 26th Asilomar Conf. Signals, Syst., Computers,* Pacific Grove, CA, Oct. 1992, pp. 940–944.

[3] ——, "The recurrent region for path metric differences in the class-2 partial-response Viterbi detector," in preparation.

[4] A. Hekstra, "An alternative to metric rescaling in Viterbi detectors," *IEEE Trans. Commun.,* vol. 37, pp. 1220–1222, Nov. 1989.

[5] R. Karabed and P. Siegel, "Matched spectral null codes for partial response channels," *IEEE Trans. Inform. Theory,* vol. 37, pp. 818–855, May 1991.

[6] ——, "Coding for higher order partial response channels," in *Coding and Signal Processing for Information Storage,* M. R. Raghuveer, S. A. Dianat, S. W. McLaughlin, and M. Hassner, Eds., in *Proc. SPIE 2605*, 1995, pp. 115–126.

[7] B. E. Moision, P. H. Siegel, and E. Soljanin, "Distance-enhancing codes for digital recording," in *Proc. 1997 IEEE Magnetic Recording Conf. (TMRC'97), IEEE Trans. Magn.,* vol. 34, pt. 1, pp. 69–74, Jan. 1998.

[8] C. Papadimitriou and K. Steiglitz, *Combinatorial Optimization: Algorithms and Complexity.* Englewood Cliffs, NJ: Prentice Hall, 1982.

[9] P. Siegel, C. Shung, T. Howell, and H. Thapar, "Exact bounds for Viterbi detector path metric differences," in *Proc. 1991 IEEE Int. Conf. Acoust., Speech, Signal Processing (ICASSP'91),* Toronto, 1991, May 14–17, 1991.

[10] *HD153068TF, 130-Mbps PRML Data Channel Processor,* Rev. 1.0, Advanced Information, Hitachi Corp., Aug. 1996.