On Near-Capacity Coding Systems for Partial-Response Channels

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Abstract — **We present a near-capacity coding system for higher-order partial-response channels, consisting of an outer set of interleaved low-density parity-check codes, an inner rate-1 shaping code, and a multistage decoder. The inner shaping code, which may be non-invertible, is designed to generate an output process similar to a binary Markov process that maximizes the mutual information for a given order. On the EPR4 channel, our system exhibits an iterative decoding threshold and a simulation BER of 10^{-5} within 0.19 and 0.33 dB, respectively, of the information-theoretic limit for a third-order input process.**

I. Introduction

Consider partial-response channels of the form, $R_t = \sum_{i=0}^{\nu} h_i X_{t-i} + N_t$, with binary-constrained input process $X_t \in \{\pm 1\}$, output R_t, impulse response $\{h_0, \ldots, h_\nu\}$, and AWGN N_t. In [1], a method to compute information rates for a finite-state Markov input process (FSMP) is used to produce the best known lower bounds on capacity. Moreover, the method can also be used to calculate the information rate for a FSMP generated from a rate k:n (i.e., k inputs symbols are mapped to n output symbols) finite-state encoder driven with i.i.d. equiprobable binary inputs. Such a rate can be achieved by concatenating the finite-state encoder with a suitable outer parity-check code [2, 3]. (This is an extension of the discrete memoryless channel case shown by Gallager [4].)

Currently, the best general approach for designing inner and outer codes in this concatenation is that of Ma, et al. [2]. However, the use of their methods may not be feasible for higher-order partial-response channels. We present a system that uses an outer multilevel code (MLC), consisting of interleaved low-density parity-check (LDPC) codes, an inner rate-1 finite-state encoder, and a multistage decoder. In contrast to [2], our design of the inner encoder, which may be non-invertible, and our optimization of the outer code are easier and more readily applicable to higher-order channels such as EPR4, with $h(D) = (1 + D - D^2 - D^3)/2$.

II. Design of Shaping and LDPC Codes

Using an approach introduced by Gallager [4, p. 208], we design the inner shaping encoder to have an output process which closely resembles an optimized input FSMP. We construct a rate k:k encoder which may map multiple input k-tuples to a k-tuple of channel input symbols, thus yielding symbol-block probabilities that are multiples of 2^{-k}. As an example, suppose the target first-order Markov process changes sign with probability 0.7. Table 1 shows a rate 2:2 encoder designed to match the two-step trellis of this target process, where "filter" denotes the channel filter output for $h(D) = 1 - D$, and p^* and p_{enc} are the target and encoder distributions, respectively. Although this encoder is not invertible, the corresponding information rate is nevertheless achievable with an outer code [3].

start	end	in	out	filter	p^*	p_{enc}
1	1	(-1,-1), (1,-1)	(1,-1)	(2,-2)	0.49	0.5
1	2	(-1,1)	(-1,1)	(0,2)	0.21	0.25
1	2	(1,1)	(1,1)	(2,0)	0.21	0.25
2	1	(-1,-1)	(-1,-1)	(-2,0)	0.21	0.25
2	1	(1,-1)	(1,-1)	(0,-2)	0.21	0.25
2	2	(-1,1), (1,1)	(-1,1)	(-2,2)	0.49	0.5

Tab. 1: Example construction for an inner shaping encoder.

An outer MLC is incorporated to ensure reliable communication [1]. Specifically, m separate length-N LDPC codewords are interleaved prior to the inner encoder. These codewords are recovered sequentially from a multistage decoder which uses an APP detector matched to the joint trellis of the channel and the inner code. At each stage i, the detector is given decisions from previous stages, and obtains log-APP ratios for the ith message-passing decoder. Using techniques from [5], at each interleave we optimize the rate of the LDPC code based on the marginal conditional density $f(l^{(i)}|u^{(i)})$, where $u^{(i)}$ is the ith interleaved input and $l^{(i)}$ is the log-APP output of the APP detector at stage i, under the assumption that all previous stages were correct.

III. Results

We apply the design technique to the EPR4 channel by constructing 6-level coding systems at rates 0.2, 0.3, 0.4, and 0.5. All systems exhibit iterative decoding thresholds within 0.25 dB of the information-theoretic limit for a third-order input FSMP. For rate 0.5, the threshold gap is 0.19 dB, and for an LDPC code blocklength of $N = 10^6$, the system achieves a BER of 10^{-5} within 0.33 dB of the third-order limit.

References

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