WORST CASE CODE PATTERNS FOR MAGNETIC BURIED SERVOS

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A technique for determining worst case run-length-limited (RLL) code patterns for data-to-servo coupling in magnetic buried servos is presented. The problem is reduced to a general dynamic programming problem, whose solution is described. The general problem can require large amounts of computation, so a method is also developed to determine approximate worst case patterns by solving a classical dynamic programming problem using the Viterbi algorithm. An error estimate for the approximate solution is also derived. Computational results are presented for the (2,7) RLL constraint.

I. Introduction

A magnetic buried servo head positioning system, and other applications of buried servo signals were described in [1,2]. The problem of interfering signals in the servo detector during data write operations was discussed there.

We have developed a technique for finding worst case patterns for this coupling between the data write driver and servo detector. These are important in determining the attenuation required from a data write filter or digital data transformation to control worst case interference during data write operations. The technique, based on dynamic programming, is applicable to run-length-limited (RLL) (d,k) constrained codestrings which are standard in recording channels using peak detection [3]. The technique extends to any set of code strings represented by a finite-state transition diagram (FSTD).

As an example, we considered code strings which satisfy the (2,7) constraint, which forms the basis for the recording code in the IBM 3380. We calculated the power of worst case patterns at several servo frequencies and compared this to the average (2,7) power in an appropriate bandwidth around those frequencies.

II. Reduction to Dynamic Programming

The finite state transition diagram (FSTD) which generates the (2,7) constraint in NRZ notation is given in Fig. 1 below:

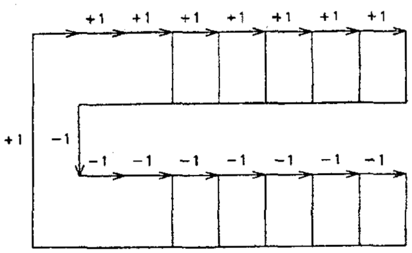

Figure 1.

For each (2,7) constrained bit string, the write driver output can be described by a square wave x(t) with amplitude levels 1 and -1.

The power of the signal x is given by:

$$P(f) = \lim_{M\to\infty} \frac{1}{TM} \left| \int_0^{TM} x(t) e^{-2\pi i f t} dt \right|^2 \qquad (1)$$

where $T=1/f_b$ (the reciprocal of the bit frequency) and M is the length of the pattern in bits. Our problem is:

> Find the (2,7) pattern x(t) (called the worst case pattern) which maximizes P(f) for a given rational multiple of the bit frequency $f=(p/q)f_b$.

We note that this is the same as maximizing the magnitude of the Fourier coefficient X(f) where

$$X(f) = \lim_{M\to\infty} \frac{1}{TM} \int_0^{TM} x(t) e^{-2\pi i f t} dt \, . \qquad (2)$$

It can be shown that the worst case power at f results from a pattern of period no greater than Nq, where N is the number of states in the finite state diagram for the code constraints. The periodic pattern is a "q-simple cycle". That is, it is a cycle of length Nq in the FSTD which begins and ends at the same state of the diagram and passes through distinct states at all intermediate multiples of q bit times. In particular, we may assume that x(t) is of finite duration t where $0 \le t \le TM$, with $TM \le Nq$. See Section IV for further discussion.

We now indicate how the maximization of X(f) can be reduced to a dynamic programming problem. See [4] for a discussion of classical dynamic programming. We re-write

$$X(f) = \frac{1}{TM} \sum_{j=0}^{M-1} x(jT) \int_{jT}^{(j+1)T} e^{-\frac{2\pi i p t}{qT}} dt \qquad (3)$$

(recall that x is constant on the interval [jT,(j+1)T)). This is a problem of the following general type.

General Dynamic Programming Problem:

Let G be an FSTD whose edges are labelled by complex numbers. Let $w_0, w_1, \ldots, w_k$ be a periodic sequence of complex numbers. For each finite path, $\gamma = e_0 e_1 \ldots e_{M-1}$, let

$$g(\gamma) = \sum_{j=0}^{M-1} l(e_j) w_j \qquad (4)$$

where $l(e_j)$ denotes the label of the jth edge, e_j, in γ. The problem is to find, for each fixed positive integer M, a path γ in G of length M which maximizes $|g(\gamma)|$.

In our case, the labels $l(e_j)$ are real and are given by x(jT) (which can be +1 or -1), and the complex numbers w_j are given by the integrals above in (3). A solution to the general problem is described in Section IV.

In order to reduce the amount of computation required, an alternative approach was developed which produces approximate worst case patterns. The original problem, which involves complex quantities, was approximated by a problem involving only real quantities. The approximate solutions require only classical dynamic programming techniques. Details are given in Section IV. An error bound, derived in Section IV, indicates that an arbitrary degree of accuracy can be achieved in the approximate solution by repeated application of the classical techniques.

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III. Computational Results

Table 1 gives approximate worst case patterns for the (2,7) constraint at a range of servo frequencies. The phase used in the calculations was $\phi=0$. See Section IV for a discussion of the choice of phase ϕ in the approximate solution. The code clock frequency $f_b=48$ MHz.

Servo Frequency (fraction)	Servo Frequency (MHz)	Period (bits)	Worst Case (2,7) Pattern (run lengths)
3f/32	4.5	16	6 5 5
f/12	4.0	6	6
7f/96	3.5	48	7 7 7 7 7 7 6
f/16	3.0	8	8
5f/96	2.5	144	8888338888338888338883 3
f/24	2.0	36	8 3 3 8 3 3 8
f/32	1.5	16	8 3 5
f/48	1.0	144	8383383833838338383383833838
f/96	0.5	48	8 3 8 3 8 3 8 3 4

Table 1

Notes:

1. Run lengths represent the number of NRZI zeros (0) plus the ending NRZI one (1). For example, 6 5 5 refers to the NRZI pattern 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1.

2. The period refers to the period of the (2,7) NRZI code pattern. The write current waveform associated to the pattern will have twice the period shown if the number of runs in the pattern is odd, and the same period if the number of runs is even. For example, repetition of the pattern 6 gives a waveform which is +1 for 6 bit times, then -1 for 6 bit times, and so on. The waveform period is 12 bits, twice the period of the (2,7) pattern. In fact, all the frequencies except f/48 have an odd number of runs in the pattern, so the waveform will have twice the period shown in the table.

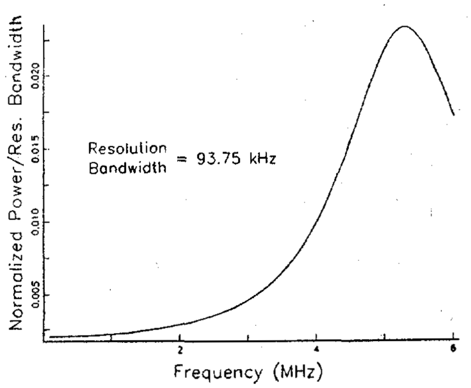

Figure 2.

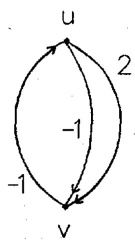

Figure 3.

Table 2 compares the power in decibels (db) of worst case patterns and the ideal (2,7) code spectrum to the unit power sinusoid at various frequencies. The resolution bandwidth used in these calculations was 93.75 kHz, corresponding to the use of 512 points in the calculations of the discrete Fourier transforms in the 48 MHz bandwidth. Figure 2 shows the theoretical, or ideal, (2,7) code power spectrum in a 6 MHz bandwidth [7]. The approximate worst case patterns have from 17 to 22 db more power in the 93.75 kHZ bandwidth at these servo frequencies than the ideal spectrum, which represents the spectrum of a "typical" (2,7) pattern.

Servo Frequency (fraction)	Servo Frequency (MHz)	Power (db) Worst Case Patterns	Power (db) Ideal (2,7) (in 93.75 kHz)
3f/32	4.5	-1.02	-17.87
f/12	4.0	-0.91	-19.86
7f/96	3.5	-0.99	-21.66
f/16	3.0	-0.91	-23.18
5f/96	2.5	-3.99	-24.45
f/24	2.0	-5.35	-25.39
f/32	1.5	-7.37	-26.10
f/48	1.0	-7.27	-26.60
f/96	0.5	-7.68	-26.88

Table 2

For an example of a pattern worse than the one found by the $\phi=0$ approximation, consider $f_b/48$. The pattern y found has $|Y(f_b/48)|=0.286976$, whereas the 240 bit pattern x given by the runs

83834 83833 83833 83833 838

has $|X(f_b/48)|=0.287306$. Note that $|Y|/|X|=0.998851$, in agreement with the error bound discussed in the next section.

IV. Theoretical Results

General dynamic programming problem: Exact solution

Let A(M,i,j) be the set of paths of length M which start at vertex i and end at vertex j. Suppose that $\gamma=e_0 \ldots e_{M-1}$ maximizes $|g(\gamma)|$ on A(M,i,j). See Eq. (4). Let $\zeta=e_0 \ldots e_{M-2}$ and let j* be the terminal vertex of e_{M-2}. It is not necessarily true that ζ maximizes $|g|$ on A(M-1,i,j*) even in the case where both the labels $l(e_k)$ and the weights w_k are real. For example, consider the FSTD in Fig. 3 with $w_k=1$, for all k, and M=2, i=j=u.

This contrasts with classical dynamic programming where the labels and weights are real and one is trying to maximize g instead of $|g|$. There it is true that if γ maximizes g on A(M,i,j), then ζ maximizes g on A(M-1,i,j*).

Nevertheless, something can be said about ζ in the general case. Namely, $g(\zeta)$ is an extreme point of the convex hull of g(A(M-1,i,j)).

So, one can modify the classical dynamic programming technique by saving, as survivor sequences for each initial state i and terminal state j, only those ζ such that $g(\zeta)$ is an extreme point of the convex hull of g(A(M,i,j)). An algorithm for computing extreme points in the plane is contained in [8]. This solution contrasts with classical dynamic programming (such as the Viterbi algorithm), in which one saves only 1 survivor path for each i and j.

General dynamic programming problem: Approximate solution

Let ϕ be a fixed angle. Then, assuming that x(t) is of finite duration, $0 \leq t \leq TM$, and viewing X(f) as a real 2-dimensional vector, we have:

$$h(x) = X(f) \cdot (\cos\phi, \sin\phi) = \frac{1}{TM}\int_0^{TM} x(t)\cos\left(\varphi + \frac{2\pi pt}{pT}\right)dt \,. \quad (5)$$

The left-hand side is the projection of X(f) onto a ray through the origin at angle ϕ. See Fig. 4.

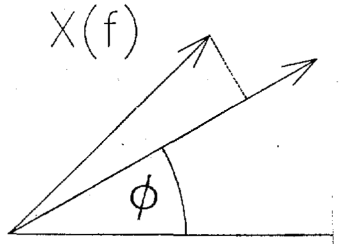

Figure 4.

The right-hand side is the correlation of x(t) with a cosinusoid of phase ϕ and frequency f. If ϕ were the argument of X(f), then (5) would give exactly $|X(f)|$. Unfortunately, the value of arg X(f) is not known in advance. So, we just choose $\phi=0$. Observe, by cyclically shifting x an integral multiple of bit times, that maximizing (5) with $\phi=0$ yields the same solution as maximizing (5) with $\phi=2\pi pj/q$, j=0,1,..., q-1. If q is large, then the $\phi=0$ solution gives a good approximation to the solution of the original problem, as indicated by the error bound below. Maximizing (5) with respect to a quantized set of phases, say $\phi_k=2\pi pk/qN$, k=0,...,N-1, gives improved estimates. The error E(p,q,N) in the estimate of the maximum $|X(f)|$ is obtained as follows: Let x be the worst case pattern and let $|X(f)|$ be the magnitude of its Fourier coefficient at f. Let y be the pattern found by the approximation using N quantized phases, and let $|Y(f)|$ be the associated magnitude. Then,

$$|X(f)|\cos\theta/2 \leq |X(f)|\cos\alpha \leq |Y(f)| \leq |X(f)| \qquad (6)$$

and

$$E(p,q,N) = |X(f)| - |Y(f)| \leq |X(f)|(1-\cos\alpha) \qquad (7)$$

$$\leq |X(f)|(1-\cos(\theta/2)) = |X(f)|(1-\cos(2\pi p/qN)) .$$

See Fig. 5. As $N\to\infty$, E(p,q,N) approaches 0. If q is large, N need not be very large to get good approximate worst case patterns. For example, if q=48 and N=1, $E(1,48,1) \leq 0.002\ |X(f_b/48)|$.

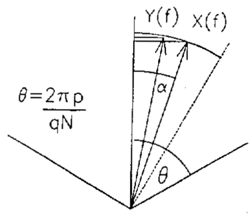

Figure 5.

The approximate problem of maximizing (5) reduces to a dynamic programming problem by rewriting the integral as:

$$h(x) = \frac{1}{M}\sum_{j=0}^{M-1} x(jT)\left[\cos\left(\varphi + \frac{2\pi pj}{q}\right)\int_0^1 \cos\frac{2\pi ps}{q}\,ds \right. \quad (8)$$

$$\left. - \sin\left(\varphi + \frac{2\pi pj}{q}\right)\int_0^1 \sin\frac{2\pi ps}{q}\,ds\right] .$$

This fits into the framework of the general dynamic programming problem mentioned above, with the special feature that both the labels of the edges (x(jT)=+1 or -1) and the numbers w_j (the quantity in brackets above) are real numbers. If we were maximizing h(x), instead of its absolute value, this would be the familiar classical dynamic programming problem, from which the Viterbi decoding algorithm is derived [5].

Since we maximize $|h(x)|$, we must solve instead two classical dynamic programming problems - namely, maximize both h(x) and -h(x), or equivalently, separately maximize and minimize h(x). (See [6] for another application of classical dynamic programming techniques to computation of worst case patterns for intersymbol interference in amplitude detection channels.)

Periodic worst case patterns

The worst case pattern can be viewed as a path which gives the largest value of $|g(\gamma)|$ per edge, irrespective of length M. So, we want to maximize

$$|g(\gamma)|/M \qquad (9)$$

Since M is typically very large in practical applications, we idealize this problem and maximize instead:

$$\lim_{M\to\infty} |g(\gamma_M)|/M \qquad (10)$$

over the set of infinite paths γ. Here, γ_M means the truncated path $\gamma_M=e_0 \ldots e_{M-1}$.

Assume initially that all of the w_i are equal to 1. Then in this case the maximum of (10) is achieved by a simple periodic cycle in the graph, that is, a closed path $e_0 \ldots e_{M-1}$, all of whose edges are distinct, repeated infinitely. This can be seen by noting that there are simple periodic cycles which come arbitrarily close to achieving the maximum of (10) (we omit the proof for reasons of space). Since a simple periodic cycle has length at most N, where N is the number of vertices in the FSTD, this gives a finite solution to the idealized problem in this special case.

In general, the sequence of w_i is periodic of some period q, such as the powers of $e^{-2\pi ip/q}$. We can reduce this to the previous special case by replacing the original FSTD G by a new FSTD H. The vertices of H are {(i,k): i=1,...,N and k=1,...,q}. For each edge e in G, from vertex i to vertex j, there are q edges in H, from (i,k) to (j, k+1 (mod q)), k=1,...,q. The corresponding edge labels in H are $l(e)w_k$. Application of the special case above to H yields the general case.

From these considerations, we see that there exists a path in G which maximizes (10) and is generated by a simple periodic cycle of length at most Nq. The dynamic programming problem can therefore be solved in a finite number of steps.

V. Conclusions

Methods for computing worst case and approximate worst case RLL code patterns for magnetic buried servo have been described. The methods were applied to the (2,7) RLL constraint. For the range of servo frequencies considered, the worst case patterns were found to have approximately 20 db greater power than an "average" (2,7) pattern in a 100 kHz bandwidth about the servo frequency.

References

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