Dimensions of Channel Coding: From Theory to Algorithms to Applications

Special Issue Dedicated to the Memory of Alexander Vardy

Alexander Vardy (1963 – 2022)

I. Introduction

THIS special issue of the IEEE Journal on Selected Areas in Information Theory is dedicated to the memory of Alexander Vardy, a pioneer in the theory and practice of channel coding. His ground-breaking contributions ranged from unexpected solutions of long-standing theoretical conjectures to ingenious decoding algorithms that broke seemingly insurmountable barriers to code performance. Inspired not just by the mathematical beauty of coding theory but also by its engineering utility, Alex developed novel coding techniques that have had a profound impact on modern information technology. At the same time, his innovations have left their imprint on other scientific disciplines, such as information theory, computer science, and discrete mathematics.

The goal of this issue is to celebrate Alex's expansive scientific and engineering legacy, in all its reach and richness. We have assembled a collection of 25 outstanding original research papers on topics inspired by Alex's diverse body of work. The papers highlight the lasting influence of Alex's discoveries and reflect, in part, the vast number of dimensions of channel coding that his research spanned: combinatorial coding theory (designs, tilings, lattices, and sphere packings); constrained codes (constructions and bounds); codes and complexity theory (hardness proofs and minimal representations); algebraic error-correcting codes (constructions and bounds); codes on graphs and trellises (classification, complexity, and constructions); decoding algorithms (soft-decision decoding, list decoding, and iterative decoding); polar codes (constructions, decoding, and performance analysis); efficient hardware implementation architectures for coding algorithms; and applications of coding to computer memories, data storage devices, communication networks, distributed computing and storage, on-chip interconnects, private and secure data communication and retrieval, and DNA-based storage.

Relatively early in his career, Alex published a number of remarkable results in sole-authored journal papers on efficient decoding algorithms for classical codes and lattices, such as the Nordstrom-Robinson code [9] and the hexacode, Golay code, and Leech lattice [14]. He also individually published record-breaking dense sphere packings [13], [29], starting with a new sphere packing in 20 dimensions that represented the first improvement in 25 years in dimensions $n \leq 29$. A ground-breaking proof of the intractability of computing the minimum distance of a code resolved a tantalizing, nearly 20-year-old conjecture [25]. Alex was also a uniquely talented and generous collaborator, and he shared his creative and deep insights with a large group of co-authors that included graduate students, postdoctoral researchers, faculty from his home institutions and from other universities, and industrial researchers. Many of the guest editors of this special issue fall into one or more of these co-author categories, and the entire guest editorial team considers it a privilege to have had the opportunity to work with and learn from a true genius who gave so much both professionally and personally. Tragically, we lost Alex to a premature death on March 11, 2022, while he was still very much at the height of his powers. The publication of this special issue, a symbol of our affection, admiration, and gratitude, honors the 60th anniversary of his birth on November 12, 1963.

II. Professional Biography

Alexander Vardy earned his B.Sc. (summa cum laude) in Electrical Engineering from the Technion-Israel Institute of

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Technology, in 1985, and Ph.D. in Electrical Engineering – Systems from Tel Aviv University, Israel, in 1991. From 1985 to 1990 he served as a Senior Research & Development Engineer with the Israeli Air Force, where he worked on electronic countermeasures systems and algorithms. During the years 1992 and 1993 he was a Visiting Scientist at the IBM Almaden Research Center, in San Jose, California. From 1993 to 1998, he was with the University of Illinois at Urbana-Champaign (UIUC), first as an Assistant Professor, then as an Associate Professor in the Coordinated Science Laboratory (CSL), the Department of Electrical and Computer Engineering, and the Department of Mathematics.

Alex joined the faculty at the University of California San Diego (UCSD) in 1998 as a Professor in the Department of Electrical and Computer Engineering, with joint appointments in the Department of Computer Science and the Department of Mathematics. He had affiliations with three UCSD research centers: the Center for Memory and Recording Research (CMRR), the Center for Wireless Communications (CWC), and the Qualcomm Institute/California Institute for Telecommunications and Information Technology (QI/Calit2). In 2013, Alex was named the inaugural holder of the Jack Keil Wolf Endowed Chair in Electrical Engineering at UCSD. While on sabbatical leave from UCSD, Alex held long-term visiting appointments with the Centre National de la Recherche Scientifique (CNRS), Sophia-Antipolis, France, the École Polytechnique Fédérale de Lausanne (EPFL), Switzerland, and the Technion, Israel. Beginning in 2015, he was a regular visitor to Nanyang Technological University (NTU) and the National University of Singapore (NUS) in Singapore.

Alex served in several positions on the editorial board of the IEEE TRANSACTIONS ON INFORMATION THEORY. He was an Associate Editor for Coding Theory during 1995 to 1998, Editor-in-Chief from 1998 to 2001, and an Executive Editorial Board member from 2010 to 2013 and from 2014 to 2019. He was Guest Editor for the 1996 special issue on Codes and Complexity. Alex was also an Editor for the SIAM JOURNAL ON DISCRETE MATHEMATICS from 2003 to 2008. He served as an elected member of the Board of Governors of the IEEE Information Theory Society for five terms, spanning the periods from 1998 to 2006 and from 2011 to 2016.

Alex received the Levi Eshkol and the Rothschild Fellowships in 1990 and 1992, respectively, an IBM Invention Achievement Award in 1993, and NSF Research Initiation and CAREER awards in 1994 and 1995. In 1996, he was appointed Fellow in the Center for Advanced Study at the University of Illinois, and received the Xerox Award for Faculty Research. In the same year, he became a Fellow of the David and Lucile Packard Foundation. In 2005, he received the Fulbright Senior Scholar Fellowship.

Alex presented keynote plenary talks at several major international conferences: the 1997 Annual ACM Symposium on the Theory of Computing (STOC), the 2004 SIAM Conference on Discrete Mathematics, the 2006 IEEE International Symposium on Information Theory (ISIT), the 2018 International Conference on Signal Processing and Communications (SPCOM), and the 2018 IEEE International Symposium on Information Theory and Its Applications (ISITA).

Alex was co-recipient with Ralf Koetter of the 2004 IEEE Information Theory Society Paper Award for the paper "Algebraic Soft-decision Decoding of Reed-Solomon Codes." He shared the Best Paper Award at the 2005 IEEE Symposium on Foundations of Computer Science (FOCS) with his Ph.D. student Farzad Parvaresh for their paper "Correcting Errors Beyond the Guruswami-Sudan Radius in Polynomial Time." In 2017, he was co-recipient with Ido Tal of the IEEE Communications Society & Information Theory Society Joint Paper Award for their 2015 paper "List Decoding of Polar Codes." The paper forms the basis for Tal and Vardy's patent on "ECC polar coding and list decoding methods and codecs," U.S. Patent No. 9,503,126, which plays a key role in the 5G New Radio standard. With his Ph.D. student Hanwen Yao and postdoctoral researcher Arman Fazeli, Alex shared the 2021 IEEE Jack Keil Wolf ISIT Student Paper Award for their paper "A Deterministic Algorithm for Computing the Weight Distribution of Polar Codes."

Alex was named a Fellow of the Institute of Electrical and Electronics Engineers (IEEE) in 1999 for "contributions to the theory and practice of channel coding." He became a Fellow of the Association for Computing Machinery (ACM) in 2017 for "contributions to the theory and practice of error-correcting codes and their study in complexity theory."

III. OVERVIEW OF RESEARCH

The following presents a concise overview of Alex's research published in more than 100 papers appearing in journals on information theory, communications, mathematics, computing, electronics, and signal processing. Some key conference papers are also references. It is organized chronologically into five time periods associated with his primary institutional affiliations.

Tel Aviv University (1987 – 1991)

Alex's graduate studies at Tel Aviv University under the supervision of Yair Be'ery and Jakov Snyders culminated in his Ph.D. dissertation titled "Coset decoding: structures and algorithms." His research led to the publication of six journal papers, including [1], [2], and four ground breaking papers on maximum-likelihood soft-decision decoding of BCH codes, Reed-Solomon codes, the Golay code, and the Leech lattice [3], [4], [5], [6].

IBM (1991 – 1993)

After finishing his Ph.D., Alex worked for a short period at IBM Haifa and then received a postdoctoral appointment as a Visiting Scientist in the Signal Processing and Recording group at the IBM Almaden Research Center in San Jose, California. During this period he began a series of works on binary codes, culminating in papers on a variety of topics: perfect codes and tilings [7], [20], [27], uniqueness of the optimal Best code and certain related optimal codes [10], and bounds on the size of codes with minimum distance three [15].

While continuing his work on efficient decoding of the Golay code and Leech lattice [8], Alex started new directions of research relating to coding for data storage, a theme that would continue throughout his career. Work initiated during this period with IBM-affiliated researchers including Mario Blaum, Jehoshua Bruck, Ron Roth, Paul Siegel, and Glenn Sincerbox eventually resulted in publications on higher-order spectral null codes for magnetic and optical recording [11], multidimensional modulation codes and interleaving schemes for holographic storage [17], [26], and MDS array codes [18] for magnetic tapes and RAID systems.

UIUC (1993 – 2000)

Alex assumed his first academic position at UIUC in 1993, and during the next five years he achieved a series of breakthroughs that firmly cemented his position as a pioneer in channel coding. Pursuing a line of research initiated towards the end of his appointment at IBM Almaden, Alex developed his ingenious density-doubling construction of the densest known sphere-packing in 20 dimensions [13]. This triggered the work of Conway and Sloane on antipode constructions, resulting in new densest known packings in dimensions 22 and 44–47. Alex then showed that the density-doubling construction can be extended to higher dimensions, resulting in new record-breaking dense packings in dimensions 27–30 [29].

During this period, Alex published a collection of important papers addressing a topic of great interest at the time: the graphical structure and trellis complexity of block codes. In a striking set of three papers with his master's student Alec LaFourcade, he proved that asymptotically good codes have infinite trellis complexity [12], developed improved general lower bounds on trellis complexity [16], and presented an algorithm producing the optimal sectionalization of a code trellis [19]. Alex also developed minimal tail-biting trellises for the binary Golay code and other codes of interest [31], a topic developed in more generality in a subsequent work with Ralf Koetter [43]. Other results in this vein included a proof of optimality of the minimal trellis for Viterbi decoding [22] and an upper bound on the trellis complexity of lattices obtained with postdoctoral researcher Vahid Tarokh [24]. Alex also demonstrated a strong connection between minimal trellises and binary decision diagrams for codes viewed as Boolean functions [33]. Some of Alex's key results in this area were compiled in a classic monograph titled "Trellis Structure of Codes" that appeared as a chapter in the two-volume HANDBOOK OF CODING THEORY [28].

In 1997, Alex resolved the long-standing 1978 conjecture of Belekamp, McEliece, and van Tilborg regarding the NP-hardness of computing the minimum distance of a binary linear code and the NP-completeness of the corresponding decision problem [25]. A follow-up collaborative work addressed the complexity of other fundamental problems in coding theory [35].

Pursuing his interest in soft-decision decoding, Alex published two papers on generalized minimum distance decoding of codes and lattices, including one with his Ph.D. student Dakshi Agrawal [21], [34]. He also studied joint equalization and coding for intersymbol interference channels [23]. In another historic collaboration with Ralf Koetter, Alex achieved a major conceptual breakthrough in the soft-decision decoding of Reed-Solomon codes, long a holy grail of coding theorists, resulting in their award-winning paper [44]. The algebraic list-decoding algorithm, now known as the Koetter-Vardy algorithm, built upon the bivariate interpolation techniques introduced by Guruswami and Sudan, cleverly converting probabilistic reliability information into a set of interpolation points and multiplicities. Alex also started working on graph-based codes and iterative decoding, providing important insights into the characterization and limitations of codes with cycle-free Tanner graphs with Tuvi Etzion and his Ph.D. student Ari Trachtenberg [32], a signal space characterization of iterative decoding [37], and an analysis of phase trajectories of turbo decoding [36].

UCSD (1998 – 2022)

Alex joined the faculty at UCSD in 1998. He spent the 1998-1999 year on sabbatical, and formally ended his affiliation with UIUC in 2000. During this transition period, he made further contributions to bounds on binary codes [38] and lattice codes [30]. The highly cited semitutorial paper [40] with postdoctoral researcher Erik Agrell, Thomas Eriksson, and Ken Zeger provided a unified framework for closest point search methods for lattices lacking a regular structure, as well as a new efficient algorithm.

At UCSD, Alex made further significant contributions to topics he had worked on previously, including multidimensional interleaving schemes [39], the dynamics of turbo decoding [41], [54], tilings and sphere packings [42], [45], [47], [55], [69], [76], and decoding of Reed-Solomon codes, including NP-hardness of maximum-likelihood decoding [48] and improvements to algebraic list decoding developed with his Ph.D. student Jun Ma [56], [60]. In a surprising paper, he presented an asymptotic improvement of the Gilbert-Varshamov bound for binary codes [46]. He also published his award-winning FOCS paper with his Ph.D. student Farzad Parvaresh, in which they introduced list-decoding based on multivariate interpolation and the family of Parvaresh-Vardy codes, achieving polynomial-time error-correction beyond the Guruswami-Sudan radius [51]. With postdoctoral researcher Navin Kashyap, he also studied constrained codes for optical communication channels [49], [50]. On the topic of iterative decoding of graph-based codes, Alex and postdoctoral researcher Moshe Schwartz introduced the concept of stopping redundancy and proved several general and code specific bounds [53], [57].

The introduction of network coding and subspace codes provided a new avenue of research for Alex. In [59], Alex and Tuvi Etzion developed bounds and constructions for codes in projective space, while in [67] fundamental concepts of complements and linear codes in the projective setting were addressed. With his Ph.D. student Hessam Mahdavifar, Alex proposed efficient algebraic list-decoding algorithms for subspace codes and rank-metric codes [70], [85]. Alex also developed a general construction of certain minimum storage regenerating codes for distributed storage [82] with his Ph.D. student Arman Fazeli and postdoctoral researcher Sreechakra Goparaju, as well as improved schemes for optimal repair of MDS codes [91] with his master's student Ameera Chowdhury. In a paper appearing in this issue, Alex and Eitan Yaakobi demonstrate the power of coding versus replication for storage-efficient private information retrieval from distributed data servers [A12].

Another set of papers that appeared during this period had a combinatorial flavor, treating a range of topics including anticodes in graphs [52], q-analogs of Steiner systems and covering designs [61], [80], t-designs over finite fields (with Ph.D. student Arman Fazeli) [73], existence of large sets of designs (with co-advised Ph.D. student Sankeerth Rao) [86], and group testing [102].

At UCSD, Alex was Eitan Yaakobi's Ph.D. co-advisor with Jack Wolf and Paul Siegel. Motivated by applications to flash memory, he published several papers on write-once-memory (WOM) codes, rewriting codes, and wear-leveling codes [58], [64], [65], [71], as well as a paper on multidimensional constrained codes [62].

Alex immediately recognized the importance of Arıkan's revolutionary introduction of capacity-achieving polar codes in 2009. In an early work [75], he developed non-linear kernels with improved error exponent. He then tackled two major obstacles to the practical realization of polar codes' potential, focusing on two problems: how to construct the codes efficiently, and how to improve their performance at moderate block lengths under successive cancellation decoding. With postdoctoral researcher Ido Tal, he provided ground-breaking solutions to both problems in a pair of his most highly cited papers. In [68], they provided methods for approximating a bit channel between degraded and upgraded quantized versions, resulting in a practical construction algorithm whose complexity is linear in the code length n, with a constant factor depending on a tunable fidelity parameter. In the award-winning paper [74], they introduced their successive cancellation list decoder, which still stands as the standard against which all other decoders are compared. Both the construction method and the list decoder are widely referred to as "Tal-Vardy" algorithms.

The theory and practice of polar codes remained a major focus of Alex's research for the remainder of his career. In a series of papers with Warren Gross and others, Alex proposed efficient algorithms and hardware implementations for encoding and decoding [66], [72], [78], [79], [98]. With Hessam Medhavifar, Alex showed how to use polar codes to achieve the secrecy capacity of the wiretap channel [63], and in a subsequent review paper with postdoctoral researcher Himanshu Tyagi he revisited the topic of coding for wiretap channels using universal hashing techniques [77]. With Arman Fazeli and his Ph.D. student Hanwen Yao, he studied list decoding of polarization-adjusted convolutional (PAC) codes [89], and also provided a deterministic algorithm for finding the weight distribution of polar codes in an award-winning conference paper [90]. Alex, in collaboration with Arman Fazeli and others, also made important contributions to the design of polar codes with improved scaling exponent [92], [101] and the application of polarization and polar coding to channels with deletions [81], [94], [97].

NTU and NUS (2015 – 2022)

Alex began a long-term collaboration with several researchers in Singapore, notably Yeow Meng Chee and Han Mao Kiah, when he spent a one-year sabbatical at NTU from 2015 to 2016. Combining their mutual interests in discrete mathematics, combinatorial coding, and applications of coding to circuit interconnects and data storage, they, along with other co-authors, produced a series of papers on combinatorial topics such as explicit constructions of Baranyai partitions for quadruples [93], enumeration and bounds for k-decks [99], and domination mappings into the Hamming ball [100], as well as a wealth of application-oriented papers on cooling codes for thermal management of network interconnects [83], [88], efficient WOM codes and other codes for endurance-limited memories [87], [96], racetrack memory [A25], [84], [95], and DNA storage [A21], [A22]. Three of these papers [A21], [A22], and [A25] appear in this special issue. Further fruitful collaboration with colleagues in Singapore was cut short by the COVID-19 pandemic and Alex's untimely death.

IV. Contributed Papers

The twenty-five papers in this special issue reflect the diversity of topics in channel coding that Alex's research deeply influenced. The papers are grouped into five general subject areas: theory of codes and lattices, decoding of algebraic and graph-based codes, coding for networks and distributed systems, polar codes and applications, and coding for memories and in-memory computing.

Theory of codes and lattices

The article "The Voronoi region of the Barnes-Wall lattice Λ_{16}," by Pook-Kolb et al. [A1], describes in detail the Voronoi region of the Barnes-Wall lattice Λ_{16}, including its vertices, relevant vectors, and symmetry group. The exact value of the quantizer constant of the lattice is provided for the first time.

The article "Estimating the sizes of binary error-correcting constrained codes,"by Rameshwar and Kashyap [A2], investigates binary constrained codes designed to withstand bit-flip errors and erasures. Fourier analysis techniques are used to analyze the sizes of constrained subcodes of linear codes, and an extended version of Delsarte's linear program is used to derive strong upper bounds on the maximum sizes of constrained codes capable of correcting a fixed number of combinatorial errors or erasures that numerically surpass the generalized sphere packing bounds of Fazeli, Vardy, and Yaakobi.

The article "New bounds on the size of binary codes with large minimum distance," by Pang et al. [A3], presents new lower and upper bounds on the size of a binary code of length n and minimum distance d. The authors show a sequence of cyclic codes that improve upon Delsarte-Goethals codes and provide the best lower bound in a particular range of d, and they develop an upper bound that scales polynomially in n and improves upon the Barg-Nogin upper bound in a particular high-minimum distance regime.

The article "An efficient strategy to count cycles in the Tanner graph of quasi-cyclic LDPC codes," by Gómez-Fonseca et al. [A4], presents an efficient strategy to enumerate cycles of length less than twice the girth in the Tanner graph of a quasi-cyclic low-density parity-check (LDPC) code. The approach, which works for both regular and irregular protographs, uses properties of the powers of a certain polynomial adjacency matrix that describes the protograph lifting process.

Decoding of algebraic and graph-based codes

The article "Fast and low-complexity soft-decision generalized integrated interleaved decoder," by Tang and Zhang [A5], introduces soft-decision decoding algorithms for RS/BCH-based generalized integrated interleaved (GII) codes. Different methods of integrating the Chase algorithm into the conventional hard-decision decoding process are analyzed and compared. A new polynomial selection scheme, formulas for decoder error performance, and low-complexity hardware architectures are presented.

The article "A novel Chase Koetter-Vardy algorithm," by Wu and Ma [A6], combines the Chase flipping algorithm with both Guruswami-Sudan (GS) and Koetter-Vardy (KV) list-decoding algorithms. The Chase GS decoding algorithm can list-correct errors up to the Johnson bound for the shortened code that does not contain Chase flipping symbols. The proposed Chase KV decoding algorithm significantly outperforms the original KV decoding algorithm under similar complexity. Additionally, it is shown that the computational complexity can be reduced significantly by considering only message polynomials with constant term equal to zero, with negligible performance degradation.

The article "Machine learning-aided efficient decoding of Reed–Muller subcodes," by Jamali et al. [A7], considers subcodes of Reed-Muller (RM) codes which have flexible rate profiles. A recently proposed recursive projection-aggregation (RPA) decoding algorithm for RM codes is extended to decode subcodes of RM codes for both hard and soft information. Also, a machine learning model is trained to prune RPA, which with negligible performance degradation results in a significant complexity reduction in the decoding algorithm.

The article "High-speed LFSR decoder architectures for BCH and GII codes," by Wu [A8], presents two high-speed linear feedback shift register (LFSR) based decoder architectures for binary BCH codes, both achieving the critical path of one multiplier and one adder. A unified LFSR-based decoder is devised for generalized integrated interleaved (GII) codes that interleave BCH or Reed-Solomon (RS) codewords. Applied to error correction of GII-BCH and GII-RS codes and erasure corection of GII-RS codes, the decoder achieves the same critical path of one multiplier, one adder, and one multiplexer.

The article "A graph-based soft-decision decoding scheme for Reed-Solomon codes," by Lee et al. [A9], presents a new graph-based iterative soft decoding algorithm for Reed-Solomon (RS) codes based on the binary representations transferred from the parity-check matrices for RS codes. Using dynamic scheduling based on nested-polling residual belief propagation together with a bit flipping technique, the soft decoder gives performance close to the maximum-likelihood bound for various block lengths.

The article "Error propagation mitigation in sliding window decoding of spatially coupled LDPC codes," by Zhu et al. [A10], examines decoder error propagation in spatially coupled low-density parity-check (LDPC) codes under low-latency conditions, where the use of smaller decoding windows leads to occasional severe error propagation. To address this, a multi-state decoder model is proposed, and two methods, check node (CN) doping and variable node (VN) doping, are introduced to mitigate the issue. An adaptive approach dependent on a noiseless binary feedback channel is also suggested to further enhance performance. Simulation results demonstrate that CN and VN doping significantly improve performance within the desired operating range, with a small rate loss, while adaptive doping further enhances this improvement.

Coding for networks and distributed systems

The article "Capacity of locally recoverable codes," by Mazumdar [A11], studies a less explored property of locally recoverable codes (LRCs) – codes originally motivated by distributed storage applications in which any coordinate of a codeword can be recovered by accessing only a small subset of other coordinates – namely, their performance on stochastic channels. The paper considers the use of LRCs on input-symmetric discrete memoryless channels and gives a tight characterization of their gap to Shannon capacity.

The article "Private information retrieval without storage overhead: Coding instead of replication," by Vardy and Yaakobi [A12], addresses the problem of reducing the storage overhead associated with information-theoretic private information retrieval (PIR) protocols that replicate a database on k non-communicating servers, without sacrificing privacy and complexity guarantees. Introducing the idea of coded PIR protocols, in which each server stores a coded subset of the database instead of a complete replica, the authors show that the storage overhead (the ratio of the amount of data stored on all servers to the size of the database) can be made arbitrarily close to the optimal value of 1, while essentially preserving the communication and computational complexity achieved through replication.

The article "Distributed matrix computations with low-weight encodings," by Das et al. [A13], considers problems of coded distributed matrix-vector and matrix-matrix computations, with a focus on cases with sparse input matrices. The authors propose a new coding scheme that utilizes codewords that are functions of smaller numbers of submatrices, effectively preserving the sparsity in input matrices. This scheme achieves the same level of straggler mitigation as other well-known approaches while improving various other factors, including the worker computation time and the encoding time.

The article "Securely aggregated coded matrix inversion," by Charalambides et al. [A14], considers the problem of approximating the inverse of an aggregated data matrix across client devices in a distributed system in the presence of stragglers and eavesdroppers. The idea is to approximate columns of the inverse matrix by solving multiple least squares problems using a gradient descent algorithm. This approach protects the system against stragglers and lets clients approximate the inverse locally without revealing the data to the coordinator and eavesdroppers.

The article "A revisit of linear network error correction coding," by Guang and Yeung [A15], considers linear network error correction coding when errors may occur on the edges of a communication network of which the topology is known. The paper presents a new framework for linear network error correction coding, and proves the equivalence of two other well-known approaches, which can be unified under this framework.

The article "Cache-aided communication schemes via combinatorial designs and their q-analogs," by Agrawal et al. [A16], considers a broadcast setup with a single server broadcasting information to a number of clients, each of which contains a local cache of some size. Considering a coded caching framework, a new design approach, based on binary matrices together with combinatorial designs and their q-analogs, is then proposed for reducing the subpacketization level in the coded caching problem, and for reducing file complexity in the coded MapReduce problem.

Polar codes and applications

The article "Channel coding at low capacity," by Fereydounian et al. [A17], addresses fundamental limits of optimal finite-length coding in the low-capacity regime, as well as code constructions combining state-of-the-art channel codes with repetition. The optimal number of repetitions for transmission over binary memoryless symmetric (BMS) channels, in terms of the code block length and the underlying channel capacity, is characterized, and it is shown that the polarization transform implicitly induces this optimal number of repetitions in the construction of capacity-achieving polar codes.

The article "Randomized polar codes for anytime distributed machine learning," by Bartan and Pilanci [A18], presents a novel distributed computing framework that is robust to slow compute nodes, and is capable of both approximate and exact computation of linear operations. The proposed mechanism integrates the concepts of randomized sketching and polar codes in the context of coded computation. Applications of this framework to large-scale matrix multiplication and black-box optimization are demonstrated, and numerical results from implementation on a serverless cloud computing system are provided.

The article "Graph coded Merkle tree: Mitigating data availability attacks in blockchain systems using informed design of polar factor graphs," by Mitra et al. [A19], presents a new approach using polar encoding graphs to mitigate data availability (DA) attacks in blockchain systems. In particular, a certain graph design algorithm, based on the recursive structure of polar encoders, and an algorithm to prune the resulting graph are presented. It is demonstrated that the resulting DA detection scheme improves upon comparable schemes leveraging other classes of codes, including low-density parity-check (LDPC) and Reed-Solomon (RS) codes.

The article "On the minimum weight codewords of PAC codes: The impact of pre-transformation," by Rowshan and Yuan [A20], computes the minimum distance and the error coefficient (the number of codewords with weight equal to the minimum distance) of polarization-adjusted convolutional (PAC) codes. It is shown that the convolutional pre-transformation of the polar code does not decrease the minimum distance, but may reduce the error coefficient. A low-complexity enumeration method is used to determine the reduction in the error coefficient due to the pre-transformation.

Coding for memories and in-memory computing

The article "Bee identification problem for DNA strands," by Chrisnata et al. [A21], extends the bee identification problem for DNA-based storage applications. The aim is to identify a set of codewords from the unordered noisy outputs of a multi-draw channel, in which each codeword is transmitted a fixed number of times over a noisy channel. Through a reduction of this problem to a minimum-cost flow problem on a related bipartite input-output flow network, the authors demonstrate that the problem of finding a maximum likelihood solution can be solved in cubic time in the worst case. The input-output flow networks for deletion channels and insertion channels are studied in detail.

The article "Efficient algorithms for the bee-identification problem," by Kiah et al. [A22], considers the task of identifying barcoded "bees" using an unordered set of noisy barcode measurements. For noisy measurements modeled by the binary erasure (BEC) and the binary symmetric channels (BSC), the authors reduce this bee-identification problem to finding a perfect matching and a minimum-cost matching, respectively, They obtain joint decoders for the two scenarios that run, respectively, in time quadratic and cubic in the number of "bees." They further reduce the running time by use of peeling decoders and list-decoders and study when these joint decoders fail to identify the "bees" correctly for explicit codebooks.

The article "Genomic compression with read alignment at the decoder," by Gershon and Cassuto [A23], proposes a compression scheme for genomic data represented as a collection of short noisy fragments (reads) given a reference sequence. The reference sequence is used only during decoding, thereby simplifying the encoding process. The main ingredient of the distributed source coding scheme is a multi-layer code construction. Decoding algorithms and probabilistic performance analysis are provided for the scenarios where the reference and the reads differ by substitution errors only or multiple substitution errors and a single deletion error. For the latter scenario, a new alignment metric called shift-compensating distance is introduced.

The article "On the implementation of Boolean functions on content-addressable memories," by Roth [A24], proposes a content-addressable memory (CAM) implementation of various comparison functions and all Boolean functions that map an integer into a binary bit. All the proposed mappings are shown to be optimal in that they require the smallest possible number of CAM cells.

The article "Transverse-read-codes for domain wall memories," by Chee et al. [A25], studies a new family of codes, dubbed transverse-read codes, for domain wall memories, also known as racetrack memories. These codes are designed to take advantage of a novel read mechanism that combines transverse-read and domain shifting operations. Combinatorial properties and maximal asymptotic information rates are analyzed. Constructions of transverse-read codes that can correct multiple shift errors or limited-magnitude substitution errors are presented.

The guest editors sincerely hope that this Special Issue and the contributed papers within it will serve as a tribute to Alex Vardy's remarkable career as a pioneer in coding theory. We further hope that the issue will inspire future generations of researchers to sustain the evolution of the subject of channel coding that Alex so deeply loved.

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APPENDIX: RELATED ARTICLES

[A1] D. Pook-Kolb, E. Agrell, and B. Allen, "The Voronoi region of the Barnes-Wall lattice Λ_{16}," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 16–23, 2023, doi: 10.1109/JSAIT.2023.3276897.
[A2] V. A. Rameshwar and N. Kashyap, "Estimating the sizes of binary error-correcting constrained codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 144–158, 2023, doi: 10.1109/JSAIT.2023.3279113.
[A3] J. C.-J. Pang, H. Mahdavifar, and S. S. Pradhan, "New bounds on the size of binary codes with large minimum distance," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 219–231, 2023, doi: 10.1109/JSAIT.2023.3295836.
[A4] A. Gómez-Fonseca, R. Smarandache, and D. G. M. Mitchell, "An efficient strategy to count cycles in the Tanner graph of quasi-cyclic LDPC codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 499–513, 2023, doi: 10.1109/JSAIT.2023.3315585.
[A5] Y. J. Tang and X. Zhang, "Fast and low-complexity soft-decision generalized integrated interleaved decoder," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 174–189, 2023, doi: 10.1109/JSAIT.2023.3296419.
[A6] Y. Wu and J. Ma, "A novel Chase Koetter-Vardy algorithm," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 232–244, 2023, doi: 10.1109/JSAIT.2023.3297489.
[A7] M. V. Jamali, X. Liu, A. V. Makkuva, H. Mahdavifar, S. Oh, and P. Viswanath, "Machine learning-aided efficient decoding of Reed–Muller subcodes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 260–275, 2023, doi: 10.1109/JSAIT.2023.3298362.
[A8] Y. Wu, "High-speed LFSR decoder architectures for BCH and GII codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 331–350, 2023, doi: 10.1109/JSAIT.2023.3304235.
[A9] H.-C. Lee, J.-H. Wu, C.-H. Wang, and Y.-L. Ueng, "A graph-based soft-decision decoding scheme for Reed-Solomon codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 420–433, 2023, doi: 10.1109/JSAIT.2023.3315453.
[A10] M. Zhu, D. G. M. Mitchell, M. Lentmaier, and D. J. Costello, "Error propagation mitigation in sliding window decoding of spatially coupled LDPC codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 470–486, 2023, doi: 10.1109/JSAIT.2023.3312656.
[A11] A. Mazumdar, "Capacity of locally recoverable codes," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 276–285, 2023, doi: 10.1109/JSAIT.2023.3300901.
[A12] A. Vardy and E. Yaakobi, "Private information retrieval without storage overhead: Coding instead of replication," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 286–301, 2023, doi: 10.1109/JSAIT.2023.3285665.
[A13] A. B. Das, A. Ramamoorthy, D. J. Love, and C. G. Brinton, "Distributed matrix computations with low-weight encodings," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 363–378, 2023, doi: 10.1109/JSAIT.2023.3308768.
[A14] N. Charalambides, M. Pilanci, and A. O. Hero, "Securely aggregated coded matrix inversion," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 405–419, 2023, doi: 10.1109/JSAIT.2023.3312233.
[A15] X. Guang and R. W. Yeung, "A revisit of linear network error correction coding," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 514–523, 2023, doi: 10.1109/JSAIT.2023.3317941.
[A16] S. Agrawal, K. V. Sree, P. Krishnan, A. Vaishya, and S. Kale, "Cache-aided communication schemes via combinatorial designs and their q-analogs," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 551–568, 2023, doi: 10.1109/JSAIT.2023.3320068.
[A17] M. Fereydounian, M. V. Jamali, H. Hassani, and H. Mahdavifar, "Channel coding at low capacity," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 351–362, 2023, doi: 10.1109/JSAIT.2023.3305874.
[A18] B. Bartan and M. Pilanci, "Randomized polar codes for anytime distributed machine learning," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 393–404, 2023, doi: 10.1109/JSAIT.2023.3310931.
[A19] D. Mitra, L. Tauz, and L. Dolecek, "Graph coded Merkle tree: Mitigating data availability attacks in blockchain systems using informed design of polar factor graphs," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 434–452, 2023, doi: 10.1109/JSAIT.2023.3315148.
[A20] M. Rowshan and J. Yuan, "On the minimum weight codewords of PAC codes: The impact of pre-transformation," *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 487–498, 2023, doi: 10.1109/JSAIT.2023.3312678.

[A21] J. Chrisnata, H. M. Kiah, A. Vardy, and E. Yaakobi, “Bee identification problem for DNA strands,” *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 190–204, 2023, doi: 10.1109/JSAIT.2023.3294423.

[A22] H. M. Kiah, A. Vardy, and H. Yao, “Efficient algorithms for the bee-identification problem,” *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 205–218, 2023, doi: 10.1109/JSAIT.2023.3296077.

[A23] Y. Gershon and Y. Cassuto, “Genomic compression with read alignment at the decoder,” *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 314–330, 2023, doi: 10.1109/JSAIT.2023.3300831.

[A24] R. M. Roth, “On the implementation of Boolean functions on content-addressable memories,” *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 379–392, 2023, doi: 10.1109/JSAIT.2023.3279333.

[A25] Y. M. Chee, A. Vardy, V. K. Vu, and E. Yaakobi, “Transverse-read-codes for domain wall memories,” *IEEE J. Sel. Areas Inf. Theory*, vol. 4, pp. 784–793, 2023, doi: 10.1109/JSAIT.2023.3334303.

REFERENCES

[1] A. Vardy, J. Snyders, and Y. Be’ery, “Bounds on the dimension of codes and subcodes with prescribed contraction index,” *Linear Algebra Appl.*, vol. 142, pp. 237–261, Dec. 1990.

[2] A. Vardy and Y. Be’ery, “On the problem of finding zero-concurring codewords,” *IEEE Trans. Inf. Theory*, vol. 37, no. 1, pp. 180–187, Jan. 1991.

[3] A. Vardy and Y. Be’ery, “Bit-level soft decision decoding of Reed-Solomon codes,” *IEEE Trans. Commun.*, vol. 39, no. 3, pp. 440–445, Mar. 1991.

[4] A. Vardy and Y. Be’ery, “More efficient soft decoding of the Golay codes,” *IEEE Trans. Inf. Theory*, vol. 37, no. 3, pp. 667–672, May 1991.

[5] A. Vardy and Y. Be’ery, “Maximum likelihood decoding of the Leech lattice,” *IEEE Trans. Inf. Theory*, vol. 39, no. 4, pp. 1435–1444, Jul. 1993.

[6] A. Vardy and Y. Be’ery, “Maximum-likelihood soft decision decoding of BCH codes,” *IEEE Trans. Inf. Theory*, vol. 40, no. 2, pp. 546–554, Mar. 1994.

[7] T. Etzion and A. Vardy, “Perfect binary codes: Constructions, properties, and enumeration,” *IEEE Trans. Inf. Theory*, vol. 40, no. 3, pp. 754–763, May 1994.

[8] O. Amrani, Y. Be’ery, A. Vardy, F.-W. Sun, and H. C. A. van Tilborg, “The Leech lattice and the Golay code: Bounded-distance decoding and multilevel constructions,” *IEEE Trans. Inf. Theory*, vol. 40, no. 4, pp. 1030–1043, Jul. 1994.

[9] A. Vardy, “The Nordstrom–Robinson code: Representation over GF(4) and efficient decoding,” *IEEE Trans. Inf. Theory*, vol. 40, no. 5, pp. 1686–1693, Sep. 1994.

[10] S. Litsyn and A. Vardy, “The uniqueness of the Best code,” *IEEE Trans. Inf. Theory*, vol. 40, no. 5, pp. 1693–1698, Sep. 1994.

[11] R. M. Roth, P. H. Siegel, and A. Vardy, “High-order spectral null codes: Constructions and bounds,” *IEEE Trans. Inf. Theory*, vol. 40, no. 6, pp. 1826–1840, Nov. 1994.

[12] A. Lafourcade and A. Vardy, “Asymptotically good codes have infinite trellis complexity,” *IEEE Trans. Inf. Theory*, vol. 41, no. 2, pp. 555–559, Mar. 1995.

[13] A. Vardy, “A new sphere packing in 20 dimensions” *Inventiones Math.*, vol. 121, pp. 119–133, Dec. 1995.

[14] A. Vardy, “Even more efficient bounded-distance decoding of the hexacode, the Golay code, and the Leech lattice,” *IEEE Trans. Inf. Theory*, vol. 41, no. 5, pp. 1495–1499, Sep. 1995.

[15] Y. Klein, S. Litsyn and A. Vardy, “Two new bounds on the size of binary codes with a minimum distance of three,” *Des., Codes, Cryptogr.*, vol. 6, pp. 219–228, Nov. 1995.

[16] A. Lafourcade and A. Vardy, “Lower bounds on trellis complexity of block codes,” *IEEE Trans. Inf. Theory*, vol. 41, no. 6, pp. 1938–1955, Nov. 1995.

[17] A. Vardy, M. Blaum, P. H. Siegel, and G. T. Sincerbox, “Conservative arrays: Multi-dimensional modulation codes for holographic recording,” *IEEE Trans. Inf. Theory*, vol. 42, no. 1, pp. 227–230, Jan. 1996.

[18] M. Blaum, J. Bruck, and A. Vardy, “MDS array codes with independent parity symbols,” *IEEE Trans Inf. Theory*, vol. 42, no. 2, pp. 529–542, Mar. 1996.

[19] A. Lafourcade and A. Vardy, “Optimal sectionalization of a trellis,” *IEEE Trans. Inf. Theory*, vol. 42, no. 3, pp. 689–703, May 1996.

[20] G. D. Cohen, S. Litsyn, A. Vardy, and G. Zémor, “Tilings of binary spaces,” *SIAM J. Disc. Math.*, vol. 9, no, 3, pp. 393–412, 1996.

[21] G. D. Forney and A. Vardy, “Generalized minimum distance decoding of Euclidean-space codes and lattices,” *IEEE Trans. Inf. Theory*, vol. 42, no. 6, pp. 1992–2026 Nov. 1996.

[22] A. Vardy and F. R. Kschischang, “Proof of a conjecture of McEliece regarding the expansion index of the minimal trellis,” *IEEE Trans. Inf. Theory*, vol. 42, no. 6, pp. 2027–2033, Nov. 1996.

[23] D. Yellin, A. Vardy, and O. Amrani, “Joint equalization and coding for intersymbol interference channels,” *IEEE Trans. Inf. Theory*, vol. 43, no. 2, pp. 409–425, Mar. 1997.

[24] V. Tarokh and A. Vardy, “Upper bounds on trellis complexity of lattices,” *IEEE Trans. Inf. Theory*, vol. 43, no. 4, pp. 1294–1300, Jul. 1997.

[25] A. Vardy, “The intractability of computing the minimum distance of a code,” *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1757–1766, Nov. 1997.

[26] M. Blaum, J. Bruck, and A. Vardy, “Interleaving schemes for multidimensional cluster errors,” *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 730–743, Mar. 1999.

[27] T. Etzion and A. Vardy, “On perfect codes and tilings: Problems and solutions,” in *SIAM J. Discr. Math.*, vol. 11, no. 2, pp. 205–223, 1998.

[28] A. Vardy, “Trellis structure of codes,” in *Handbook of Coding Theory*, vol. 2. Amsterdam, The Netherlands: Elsevier, 1998, pp. 1989–2118.

[29] A. Vardy, “Density-doubling, double-circulants, and new sphere packings,” *Trans. Amer. Math. Soc.*, vol. 351, no. 1, pp. 271–283, 1999.

[30] V. Tarokh, A. Vardy, and K. Zeger, “Universal bound on the performance of lattice codes,” *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 670–681, Mar. 1999.

[31] A. R. Calderbank, G. D. Forney, and A. Vardy, “Minimal tail-biting trellises: The Golay code and more,” *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1435–1455, Jul. 1999.

[32] T. Etzion, A. Trachtenberg, and A. Vardy, “Which codes have cycle-free Tanner graphs?” *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 2173–2181, Sep. 1999.

[33] J. Lafferty and A. Vardy, “Ordered binary decision diagrams and minimal trellises,” *IEEE Trans. Comput.*, vol. 48, no. 9, pp. 971–986, Sep. 1999.

[34] D. Agrawal and A. Vardy, “Generalized minimum distance decoding in Euclidean space: Performance analysis,” *IEEE Trans. Inf. Theory*, vol. 46, no. 1, pp. 60–83, Jan. 2000.

[35] R. G. Downey, M. R. Fellows, A. Vardy, and G. Whittle, “The parameterized complexity of some fundamental problems in coding theory,” *SIAM J. Comput.* vol. 29, no. 2, pp. 545–570, 2000.

[36] D. Agrawal and A. Vardy, “The turbo decoding algorithm and its phase trajectories,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 699–722, Feb. 2001.

[37] B. J. Frey, R. Koetter, and A. Vardy, “Signal-space characterization of iterative decoding,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 766–781, Feb. 2001.

[38] E. Agrell, A. Vardy, and K. Zeger, “A table of upper bounds for binary codes,” *IEEE Trans. Inf. Theory*, no. 47, no. 7, pp. 3004–3006, Nov. 2001.

[39] T. Etzion and A. Vardy, “Two-dimensional interleaving schemes with repetitions: Constructions and bounds,” *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 428–457, Feb. 2002.

[40] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, “Closest point search in lattices,” *IEEE Trans. Inf. Theory*, vol. 48, no. 8, pp. 2201–2214, Aug. 2002.

[41] L. Kocarev, Z. Tasev, and A. Vardy, “Improving turbo codes by control of transient chaos in turbo-decoding algorithm,” *Electron. Lett.*, vol. 38, no. 20, pp. 1184–1186, Sep. 2002.

[42] A. Trachtenberg and A. Vardy, “Full-rank tilings of $\mathbb{F}_2^8$ do not exist,” *SIAM J. Discr. Math.*, vol. 16, no. 3, pp. 390–392, 2003.

[43] R. Koetter and A. Vardy, “The structure of tail-biting trellises: Minimality and basic principles,” *IEEE Trans. Inf. Theory*, vol. 49, no. 9, pp. 2081–2105, Sep. 2003.

[44] R. Koetter and A. Vardy, “Algebraic soft-decision decoding of Reed-Solomon codes,” *IEEE Trans. Inf. Theory*, vol. 49, no. 11, pp. 2809–2825, Nov. 2003.

[45] M. Krivelevich, S. Litsyn, and A. Vardy, “A lower bound on the density of sphere packings via graph theory,” *Int. Math. Res. Notices*, vol. 43, pp. 2271–2279, Jan. 2004.

[46] T. Jiang and A. Vardy, “Asymptotic improvement of the Gilbert-Varshamov bound on the size of binary codes,” *IEEE Trans. Inf. Theory*, vol. 50, no. 8, pp. 1655–1664, Aug. 2004.

[47] P. R. J. Östergård and A. Vardy, “Resolving the existence of full-rank tilings of binary Hamming spaces,” *SIAM J. Discr. Math.*, vol. 18, no. 2, pp. 382–387, 2004.

[48] V. Guruswami and A. Vardy, “Maximum-likelihood decoding of Reed-Solomon codes is NP-hard,” *IEEE Trans. Inf. Theory*, vol. 51, no. 7, pp. 2249–2256, Jul. 2005.

[49] N. Kashyap, P. H. Siegel, and A. Vardy, “An application of Ramsey theory to coding for the optical channel,” *SIAM J. Discr. Math.*, vol. 19, no. 4, pp. 921–937, 2005.

[50] N. Kashyap, P. H. Siegel, and A. Vardy, "Coding for the optical channel: The ghost-pulse constraint," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 64–77, Jan. 2006.

[51] F. Parvaresh and A. Vardy, "Correcting errors beyond the Guruswami-Sudan radius in polynomial time," in *Proc. 46th Annu. IEEE Symp. Found. Comput. Sci.*, Pittsburgh, PA, USA, 2005, pp. 285–294.

[52] T. Etzion, M. Schwartz, and A. Vardy, "Optimal tristance anticodes in certain graphs," *J. Combinatorial Theory, Series A*, vol. 113, pp. 189–224, Feb. 2006.

[53] M. Schwartz and A. Vardy, "On the stopping distance and the stopping redundancy of codes," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 922–932, Mar. 2006.

[54] L. Kocarev, F. Lehmann, G. M. Maggio, B. Scanavino, Z. Tasev, and A. Vardy, "Nonlinear dynamics of iterative decoding systems: Analysis and applications," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1366–1384, Apr. 2006.

[55] G. Cohen and A. Vardy, "Duality between packings and coverings of the Hamming space," *Adv. Math. Commun.*, vol. 1, no. 1, pp. 93–97, 2007.

[56] J. Ma, A. Vardy, and Z. Wang, "Low-latency factorization architecture for algebraic soft-decision decoding of Reed–Solomon codes," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 15, no. 11, pp. 1225–1238, Nov. 2007.

[57] J. Han, P. H. Siegel, and A. Vardy, "Improved probabilistic bounds on stopping redundancy," *IEEE Trans. Inf. Theory*, vol. 54, no. 4, pp. 1749–1753, Apr. 2008.

[58] A. Jiang et al., "Storage coding for wear leveling in flash memories," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 5290–5299, Oct. 2010.

[59] T. Etzion and A. Vardy, "Error-correcting codes in projective space," *IEEE Trans. Inf. Theory*, vol. 57, no. 2, pp. 1165–1173, Feb. 2011.

[60] R. Koetter, J. Ma, and A. Vardy, "The re-encoding transformation in algebraic list-decoding of Reed–Solomon codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 2, pp. 633–647, Feb. 2011.

[61] T. Etzion and A. Vardy, "On q-analogs of Steiner systems and covering designs," *Adv. Math. Commun.*, vol. 5, no. 2, pp. 161–176, 2011.

[62] M. Schwartz and A. Vardy, "New bounds on the capacity of multidimensional run-length constraints," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4373–4382, Jul. 2011.

[63] H. Mahdavifar and A. Vardy, "Achieving the secrecy capacity of wiretap channels using polar codes," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6428–6443, Oct. 2011.

[64] E. Yaakobi, P. H. Siegel, A. Vardy, and J. K. Wolf, "Multiple error-correcting WOM-codes," *IEEE Trans. Inf. Theory*, vol. 58, no. 4, pp. 2220–2230, Apr. 2012.

[65] E. Yaakobi, S. Kayser, P. H. Siegel, A. Vardy, and J. K. Wolf, "Codes for write-once memories," *IEEE Trans. Inf. Theory*, vol. 58, no. 9, pp. 5985–5999, Sep. 2012.

[66] C. Leroux, A. J. Raymond, G. Sarkis, I. Tal, A. Vardy, and W. J. Gross, "Hardware implementation of successive-cancellation decoders for polar codes," *J. Signal Process. Syst.*, vol. 69, pp. 305–315, Jul. 2012.

[67] M. Braun, T. Etzion, and A. Vardy, "Linearity and complements in projective space," *Linear Algebra Appl.*, vol. 438, no. 1, pp. 57–70, 2013.

[68] I. Tal and A. Vardy, "How to construct polar codes," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6562–6582, Oct. 2013.

[69] T. Etzion, A. Vardy, and E. Yaakobi, "Coding for the Lee and Manhattan metrics with weighing matrices," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6712–6723, Oct. 2013.

[70] H. Mahdavifar and A. Vardy, "Algebraic list-decoding of subspace codes," *IEEE Trans. Inf. Theory*, vol. 59, no. 12, pp. 7814–7828, Dec. 2013.

[71] E. Yaakobi, H. Mahdavifar, P. H. Siegel, A. Vardy, and J. K. Wolf, "Rewriting codes for flash memories," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 964–975, Feb. 2014.

[72] G. Sarkis, P. Giard, A. Vardy, C. Thibeault, and W. J. Gross, "Fast polar decoders: Algorithm and implementation," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 5, pp. 946–957, May 2014.

[73] A. Fazeli, S. Lovett, and A. Vardy, "Nontrivial t-designs over finite fields exist for all t," *J. Combinatorial Theory, Series A*, vol. 127, pp. 149–160, Sep. 2014.

[74] I. Tal and A. Vardy, "List decoding of polar codes," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2213–2226, May 2015.

[75] N. Presman, O. Shapira, S. Litsyn, T. Etzion, and A. Vardy, "Binary polarization kernels from code decompositions," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2227–2239, May 2015.

[76] A. Fazeli, A. Vardy, and E. Yaakobi, "Generalized sphere packing bound," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2313–2334, May 2015.

[77] H. Tyagi and A. Vardy, "Universal hashing for information-theoretic security," *Proc. IEEE*, vol. 103, no. 10, pp. 1781–1795, Oct. 2015.

[78] G. Sarkis, P. Giard, A. Vardy, C. Thibeault, and W. J. Gross, "Fast list decoders for polar codes," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 2, pp. 318–328, Feb. 2016.

[79] G. Sarkis, I. Tal, P. Giard, A. Vardy, C. Thibeault, and W. J. Gross, "Flexible and low-complexity encoding and decoding of systematic polar codes," *IEEE Trans. Commun.*, vol. 64, no. 7, pp. 2732–2745, Jul. 2016.

[80] M. Braun, T. Etzion, P. R. J. Östergård, A. Vardy, and A. Wassermann, "Existence of q-analogs of Steiner systems," *Forum Math. Pi*, vol. 4, p. e7, Aug. 2016.

[81] E. K. Thomas, V. Y. Tan, A. Vardy, and M. Motani, "Polar coding for the binary erasure channel with deletions," *IEEE Commun. Lett.*, vol. 21, no. 4, pp. 710–713, Apr. 2017.

[82] S. Goparaju, A. Fazeli, and A. Vardy, "Minimum storage regenerating codes for all parameters," *IEEE Trans. Inf. Theory*, vol. 63, no. 10, pp. 6318–6328, Oct. 2017.

[83] Y. M. Chee, T. Etzion, H. M. Kiah, and A. Vardy, "Cooling codes: Thermal-management coding for high-performance interconnects," *IEEE Trans. Inf. Theory*, vol. 64, no. 4, pp. 3062–3085, Apr. 2018.

[84] Y. M. Chee, H. M. Kiah, A. Vardy, V. K. Vu, and E. Yaakobi, "Coding for racetrack memories," *IEEE Trans. Inf. Theory*, vol. 64, no. 11, pp. 7094–7112, Nov. 2018.

[85] H. Mahdavifar and A. Vardy, "Algebraic list-decoding in projective space: Decoding with multiplicities and rank-metric codes," *IEEE Trans. Inf. Theory*, vol. 65, no. 2, pp. 1085–1100, Feb. 2019.

[86] S. Lovett, S. Rao, and A. Vardy, "Probabilistic existence of large sets of designs," *J. Combinatorial Theory, Series A*, vol. 176, Nov. 2020, Art. no. 105286.

[87] Y. M. Chee, H. M. Kiah, A. Vardy, and E. Yaakobi, "Explicit and efficient WOM codes of finite length," *IEEE Trans. Inf. Theory*, vol. 66, no. 5, pp. 2669–2682, May 2020.

[88] Y. M. Chee, T. Etzion, H. M. Kiah, A. Vardy, and H. Wei, "Low-power cooling codes with efficient encoding and decoding," *IEEE Trans. Inf. Theory*, vol. 66, no. 8, pp. 4804–4818, Aug. 2020.

[89] H. Yao, A. Fazeli, and A. Vardy, "List decoding of Arıkan's PAC codes," *Entropy*, vol. 23, no. 7, p. 841, 2021.

[90] H. Yao, A. Fazeli, and A. Vardy, "A deterministic algorithm for computing the weight distribution of polar codes," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Melbourne, VIC, Australia, 2021, pp. 1218–1223.

[91] A. Chowdhury and A. Vardy, "Improved schemes for asymptotically optimal repair of MDS codes," *IEEE Trans. Inf. Theory*, vol. 67, no. 8, pp. 5051–5068, Aug. 2021.

[92] A. Fazeli, H. Hassani, M. Modelli, and A. Vardy, "Binary linear codes with optimal scaling: Polar codes with large kernels," *IEEE Trans. Inf. Theory*, vol. 67, no. 9, pp. 5693–5710, Sep. 2021.

[93] Y. M. Chee, T. Etzion, H. M. Kiah, A. Vardy, and C. Wang, "Explicit Baranyai partition for quadruples, Part I: Quadrupling constructions," *J. Combinatorial Designs*, vol. 29, no. 7, pp. 447–481, 2021.

[94] K. Tian, A. Fazeli, and A. Vardy, "Polar coding for channels with deletions," *IEEE Trans. Inf. Theory*, vol. 67, no. 11, pp. 7081–7095, Nov. 2021.

[95] Y. M. Chee et al., "Locally-constrained de Bruijn codes: Properties, enumeration, code constructions, and applications," *IEEE Trans. Inf. Theory*, vol. 67, no. 12, pp. 7857–7875, Dec. 2021.

[96] Y. M. Chee, M. Horovitz, A. Vardy, V. K. Vu, and E. Yaakobi, "Endurance-limited memories: Capacity and codes," *IEEE Trans. Inf. Theory*, vol. 68, no. 3, pp. 1599–1613, Mar. 2022.

[97] I. Tal, H. D. Pfister, A. Fazeli, and A. Vardy, "Polar codes for the deletion channel: Weak and strong polarization," *IEEE Trans. Inf. Theory*, vol. 68, no. 4, pp. 2239–2265, Apr. 2022.

[98] S. A. Hashemi, M. Mondelli, A. Fazeli, A. Vardy, J. M. Cioffi, and A. Goldsmith, "Parallelism versus latency in simplified successive-cancellation decoding of polar codes," *IEEE Trans. Wireless Commun.*, vol. 21, no. 6, pp. 3909–3920, Jun. 2022.

[99] J. Chrisnata, H. M. Kiah, S. Rao, A. Vardy, E. Yaakobi, and H. Yao, "On the number of distinct k-decks: Enumeration and bounds," *Adv. Math. Commun.*, vol. 17, no. 4, pp. 960–978, 2023.

[100] Y. M. Chee, T. Etzion, H. M. Kiah, and A. Vardy, "Domination mappings into the Hamming ball: Existence, constructions, and algorithms," *Adv. Math. Commun.*, vol. 17, no. 5, pp. 1027–1059, 2023.

[101] H.-P. Wang, T.-C. Lin, A. Vardy, and R. Gabrys, "Sub-4.7 scaling exponent of polar codes," *IEEE Trans. Inf. Theory*, vol. 69, no. 7, pp. 4235–4254, Jul. 2023.

[102] H.-P. Wang, R. Gabrys, and A. Vardy, "Tropical group testing," *IEEE Trans. Inf. Theory*, vol. 69, no. 9, pp. 6098–6120, Sep. 2023.