Flash Memory: A Multi-User Perspective

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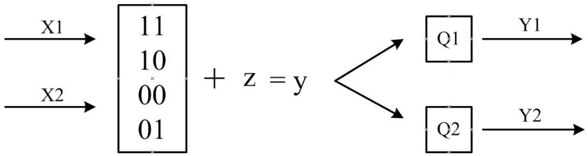
Outline

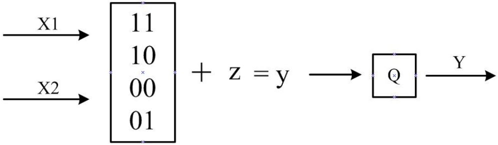
1. System Model
   - Baseline System
   - Joint Decoding System
2. Channel Model
   - Small P/E cycles
   - Large P/E cycles
3. Capacity Region Analysis
   - A 3-Reads Example
   - Multiple Reads

Baseline System



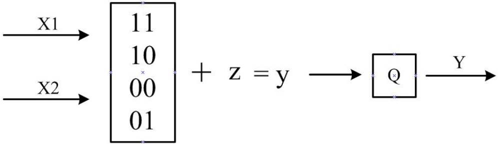
- Input: X_1 (upper page) and X_2 (lower page)
- Noise: z
- Received analog signal: y
- Quantization: Q_1 (one read) and Q_2 (two reads).
- Independent decoding performance: $R_1 \leq I(X_1; Y_1)$ and $R_2 \leq I(X_2; Y_2)$.

Joint Decoding System



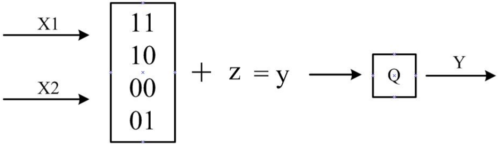
- Input: X_1 (upper page) and X_2 (lower page)
- Noise: z
- Received analog signal: y
- Quantization: Q
- Joint decoding performance:

Joint Decoding System



- Input: X_1 (upper page) and X_2 (lower page)
- Noise: z
- Received analog signal: y
- Quantization: Q
- Joint decoding performance:
  - Treating interference as noise: $R_1 \leq I(X_1; Y)$ and $R_2 \leq I(X_2; Y)$.

Joint Decoding System



- Input: X_1 (upper page) and X_2 (lower page)
- Noise: z
- Received analog signal: y
- Quantization: Q
- Joint decoding performance:
    - Treating interference as noise: $R_1 \leq I(X_1; Y)$ and $R_2 \leq I(X_2; Y)$.
    - Capacity region of this 2-user system with given input distribution:
      1) $R_1 \leq I(X_1; Y|X_2)$,
      2) $R_2 \leq I(X_2; Y|X_1)$,
      3) $R_1 + R_2 \leq I(X_1, X_2; Y)$.

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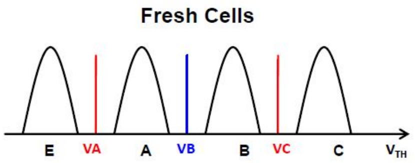
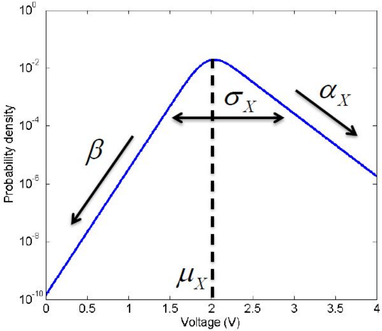
Channel Model



1. Four states: $E(11)$, $A(10)$, $B(00)$, and $C(01)$
2. Model the noise z using Normal-Laplace distribution $NL(\mu_x, \sigma_x, \alpha_x, \beta_x)$, $x \in \{E, A, B, C\}$.

The Normal-Laplace Distribution

1. An $NL(\mu, \sigma^2, \alpha, \beta)$ random variable can be expressed as $Y = \mu + \sigma W + \alpha^{-1} E_1 - \beta^{-1} E_2$.
2. PDF: $f(y) = \frac{\alpha\beta}{\alpha+\beta}\phi(\frac{y-\mu}{\sigma})[R(\alpha\sigma - (y-\mu)/\sigma) + R(\beta\sigma + (y-\mu)/\sigma)]$, where $R(x) = \frac{1-\Phi(x)}{\phi(x)}$
3. CDF: $F(y) = \Phi(\frac{y-\mu}{\sigma}) - \phi(\frac{y-\mu}{\sigma})\frac{\beta R(\alpha\sigma-(y-\mu)/\sigma)-\alpha R(\beta\sigma+(y-\mu)/\sigma)}{\alpha+\beta}$.



The Normal-Laplace Distribution

An $NL(\mu, \sigma^2, \alpha, \beta)$ distribution with various α.

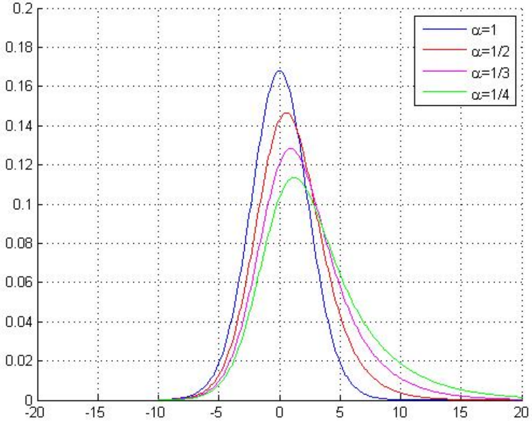
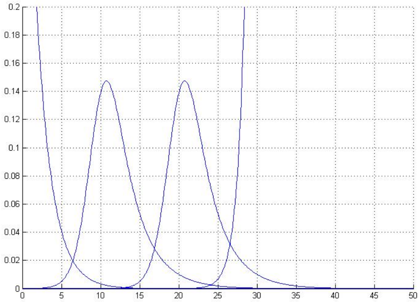


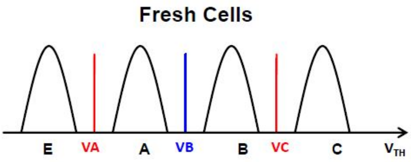
Figure: $\mu = 0$, $\sigma = 2$, $\beta = 1$

Channel model: small P/E cycles



1. State E: shifted exponential distribution, $Y_E = \mu_E + \alpha_E^{-1} E_1$.
2. State A: Normal-Laplace distribution, $Y_A = \mu_A + \sigma_A W + \alpha_A^{-1} E_1 - \beta^{-1} E_2$.
3. State B: Normal-Laplace distribution, $Y_B = \mu_B + \sigma_B W + \alpha_B^{-1} E_1 - \beta^{-1} E_2$.
4. State C: shifted exponential distribution, $Y_C = \mu_C - \beta^{-1} E_1$.
5. Ten parameters: μ_E, μ_A, μ_B, μ_C, α_E, α_A, α_B, σ_A, σ_B, β.

Channel model: large P/E cycles



For example, $E(11) \rightarrow C(01)$.

Use mixed Normal-Laplace distribution to model each state.

1. PDF of each state: $g_X(z) = \sum_{Y \in \{E,A,B,C\}} \lambda_{X,Y} f_Y(z)$, where $X \in \{E, A, B, C\}$
2. $\lambda_{X,E} + \lambda_{X,A} + \lambda_{X,B} + \lambda_{X,C} = 1$

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A 3-Reads Example

Table: Transition matrix between different states

	11	10	00	01
11	0.99	0.01	0	0
10	0	0.98	0.02	0
00	0	0	0.99	0.01
01	0	0	0	1

Table: Different decoding schemes

Capacity region	$I(X_1, X_2; Y)$	$I(X_1; Y/X_2)$	$I(X_2; Y/X_1)$
	1.9242	0.9646	0.9596
Treating interference as noise		$I(X_1, Y)$	$I(X_2, Y)$
		0.9646	0.9596
Baseline system		$I(X_1, Y_1)$	$I(X_2, Y_2)$
		0.9595	0.9571

A 3-Reads Example

Table: Different decoding schemes

Capacity region	$I(X_1, X_2; Y)$	$I(X_1; Y/X_2)$	$I(X_2; Y/X_1)$
	1.9242	0.9646	0.9596
Treating interference as noise		$I(X_1, Y)$	$I(X_2, Y)$
		0.9646	0.9596
Baseline system		$I(X_1, Y_1)$	$I(X_2, Y_2)$
		0.9595	0.9571

Observation:

- $I(X_1; Y/X_2) = I(X_1, Y) > I(X_1, Y_1)$.
- $I(X_2; Y/X_1) = I(X_2, Y) > I(X_2, Y_2)$.
- Gray mapping is good.

A 3-Reads Example

Table: Another transition matrix

	11	10	00	01
11	0.98	0.01	0	0.01
10	0	0.97	0.03	0
00	0	0	0.98	0.02
01	0	0	0	1

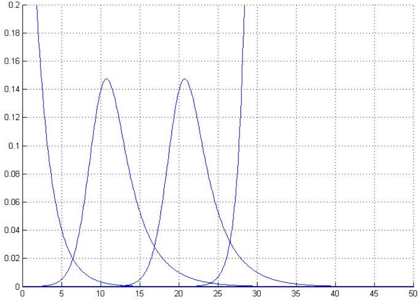
Table: Different decoding schemes

Capacity region	$I(X_1, X_2; Y)$	$I(X_1; Y/X_2)$	$I(X_2; Y/X_1)$
	1.8754	0.9311	0.9444
Treating interference as noise		$I(X_1, Y)$	$I(X_2, Y)$
		0.9310	0.9443
Baseline system		$I(X_1, Y_1)$	$I(X_2, Y_2)$
		0.9290	0.9417

- $I(X_1; Y/X_2) > I(X_1, Y) > I(X_1, Y_1)$
- $I(X_2; Y/X_1) > I(X_2, Y) > I(X_2, Y_2)$

Multiple Reads

Continuous Channel



Parameters:

- $\mu_E = 0$, $\mu_A = 10$, $\mu_B = 20$, and $\mu_C = 30$.
- $\alpha_E = 1/2$, $\alpha_A = 1/3$, and $\alpha_B = 1/3$.
- $\sigma_A = 1.5$ and $\sigma_B = 1.5$.
- $\beta = 1$.

Multiple Reads

- 3 reads: $Q[5, 15, 25]$.
- 9 reads: $Q[5-d, 5, 5+d, 15-d, 15, 15+d, 25-d, 25, 25+d]$.

Table: Different decoding schemes

Capacity region	$I(X_1, X_2; Y)$	$I(X_1; Y/X_2)$	$I(X_2; Y/X_1)$
3 reads	1.5259	0.8242	0.7060
9 reads, $d=1$	1.6079	0.8521	0.7594
9 reads, $d=2$	1.6444	0.8661	0.7816
9 reads, $d=3$	1.6410	0.8678	0.7765
9 reads, $d=4$	1.6084	0.8609	0.7513
30 reads	1.6707	0.8771	0.7967
Treating interference as noise		$I(X_1, Y)$	$I(X_2, Y)$
3 reads		0.8199	0.7017
9 reads, $d=1$		0.8485	0.7559
9 reads, $d=2$		0.8629	0.7784
9 reads, $d=3$		0.8645	0.7732
9 reads, $d=4$		0.8571	0.7475
30 reads		0.8740	0.7936
Baseline system		$I(X_1, Y_1)$	$I(X_2, Y_2)$
		0.7806	0.6869

Conclusion

- Presented a multi-user model to study fundamental limits of the flash memory.
- Calculated capacity region of the MLC flash memory.
- Compared the performance of the joint decoding scheme to the one of the baseline system.
- Extend to TLC flash memory, i.e., three-user system.

Thanks