Information-theoretic limits of two-dimensional optical recording channels

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ABSTRACT

During the past five years, advances in the information-theoretic analysis of "one-dimensional (1D)" recording channels have clarified the limits on linear densities that can be achieved by track-oriented magnetic and optical storage technologies. Channel architectures incorporating powerful codes, such as turbo codes and low-density parity-check codes, have been shown to achieve performance very close to the information-theoretic limits.

As 1D track-oriented data storage technologies reach maturity, there is increasing interest in "two-dimensional (2D)" recording technologies, such as two-dimensional optical storage (TwoDOS) and holographic storage. This paper provides an overview of some recently developed techniques for determining analytical bounds and simulation-based estimates for achievable densities of such 2D recording channels, as well as some recently proposed signal processing and coding methods that can move system performance closer to the information-theoretic limits.

Keywords: optical storage, holographic recording, two-dimensional channel, intersymbol interference channel, information theory

1. INTRODUCTION

The steady increase in storage capacity and data rate in "one-dimensional (1D)" track-oriented magnetic and optical recording devices during the past few decades has been truly remarkable. While most of this growth has been the result of improvements in the recording medium and read/write components, advances in signal processing and coding are acknowledged to have played an important role in helping to extract the greatest benefit from new component technologies.

Interest in determining the maximum possible density and data rate achievable through the application of coding and detection methods has long provided the motivation to use information-theoretic techniques to study the performance limits of digital recording channels. However, two distinguishing channel characteristics — the restriction to a binary input alphabet and the presence of memory, or intersymbol interference (ISI) — prevented the straightforward extension of classical capacity formulas. The problem of accurately computing information-theoretic limits of binary-input channels with finite memory was only recently solved, through the introduction of new analytical and simulation-based methods.

The practical importance of such information-theoretic bounds has increased dramatically over the past decade as a result of the discovery of powerful error-control techniques, such as turbo codes and low-density parity-check (LDPC) codes, as well as the introduction of the "turbo principle" into equalization, timing recovery, decoding, and detection algorithms. The application of these advanced signal processing and coding methods has enabled practical digital communication systems to achieve performance close to the capacity of the underlying channels. Considerable effort has been devoted to the design and evaluation of such techniques for possible use in magnetic and optical recording, as well.

In order to ensure the continued high rate of growth in storage capacity and data transfer rate that has characterized the progress in recording technology, interest has been turning to "two dimensional (2D)" page-oriented approaches, such as two-dimensional optical storage (TwoDOS) [1] and holographic storage [2]. In this paper, we provide a status report on techniques for determining analytical bounds and simulation-based estimates

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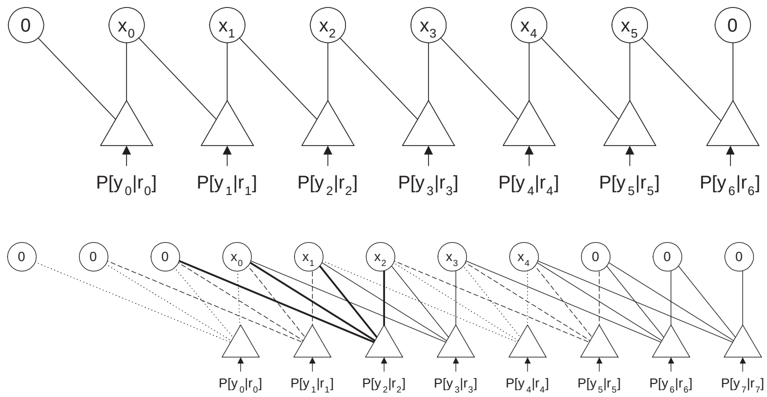

Figure 1. MPPR graphs for 1D dicode and EPR4 partial-response channels.

for achievable densities of such 2D recording channels. We also describe recently developed signal processing and coding methods intended to help realize the performance promised by information-theoretic analysis.

By way of background, we first provide a brief review of results for 1D track-oriented recording channels.

1.1. Information Rates for 1D Channels

During the past five years, new analytical bounds and simulation-based methods have been introduced that now permit the calculation of the so-called "symmetric information rate (SIR)" (corresponding to the maximum rate achievable with i.i.d., equiprobable binary inputs) as well as a sequence of lower bounds on the channel capacity (corresponding to the maximum rate achievable with arbitrary binary input sources).

Briefly, the idea underlying these methods is to estimate the channel output entropy by computing the joint probability of a long channel output realization and then invoking the Shannon-McMillan-Breimann theorem. This calculation is greatly facilitated by use of the forward recursion of the forward-backward (BCJR) algorithm [3], applied to a trellis that combines the source and channel constraints. By optimizing the source statistics for Markov sources of increasing order, one can obtain very good approximations to the channel capacity. Monte-Carlo based methods have also been used to estimate upper bounds on the input-constrained capacity. For details, the reader can consult, for example, [4], [5], [6], [7]. This approach was further generalized and applied to certain multi-track recording systems [8].

1.2. Turbo-like System Architectures

Concatenated channel architectures that use turbo-like iteration in joint detection and decoding have been shown to achieve performance close to the information-theoretic limits discussed above. In [9], [10], tools were developed for the performance analysis and design optimization of LDPC codes on 1D ISI channels with additive white Gaussian noise. By iterating between the message-passing decoder for the LDPC code and the maximum *a posteriori* (MAP) channel detector (which can be implemented using the BCJR algorithm), performance thresholds within tenths of a dB of the SIR were achieved over a wide range of code rates. Error-rate simulations for optimized LDPC codes on the dicode and EPR4 partial-response channels (with system polynomials $h_{\mathrm{dicode}}(D) = 1/\sqrt{2}(1-D)$ and $h_{\mathrm{EPR4}}(D) = 1/2(1+D-D^2-D^3)$, respectively) confirmed the analytical results.

As an alternative to the inherently serial BCJR detection algorithm, a parallel message-passing approach to detection of 1D partial-response equalization (referred to as MPPR detection) was studied in [11]. Examples of graphical structures upon which the bit-based message-passing algorithm can be performed for the dicode channel and the EPR4 channel are shown in Fig. 1.

The interconnections between the channel input nodes (represented by circles) and the "output function" nodes (represented by triangles) reflect the dependence of channel outputs upon channel inputs. Noisy channel

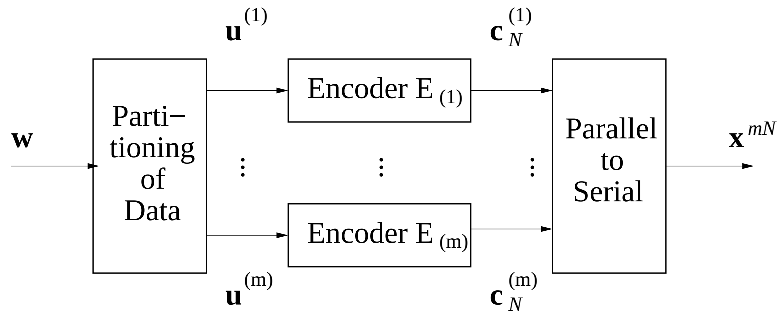

Figure 2. Multilevel encoder architecture.

outputs (or likelihood values derived from them) are provided to the "output function" nodes at the start of the detection process.

Short cycles in the bit-based trellis degrade the performance of the MPPR detector for channels with memory greater than 1, but state-based message-passing decoding based upon an underlying trellis structure was shown to provide near-optimal performance with properly chosen precoding. Performance approaching that of the BCJR detector was observed for the dicode and EPR4 channels with a moderate number of message-passing iterations. (Alternatively, the minimum cycle length of the bit-based trellis can be increased at the expense of higher graph complexity, improving the performance of the bit-based algorithm [12].)

The MPPR graph can be concatenated with the Tanner graph of an LDPC code, enabling joint message-passing detection and decoding. The performance of such a system with various message-passing schedules was evaluated empirically in [11]. In principle, this architecture should achieve performance comparable to that of the optimized LDPC-BCJR architecture described in [9], [10].

1.3. Multistage-Decoding System Architectures

Another effective 1D approach to joint equalization and decoding combines multilevel coding (MLC) (in the sense of interleaving independent code streams) and multistage decoding (MSD). An m-level encoder partitions an input block $\mathbf{w}$ into m subblocks of varying sizes. For $\ell = 1, \ldots, m$, the ℓth block is separately mapped by an encoder of rate $R_m^{(\ell)}$ into a codeword $c^{(\ell)}$ of length N. The m codewords are then interleaved and recorded. The MLC encoder architecture is shown in Fig. 2.

The multistage decoder recovers the recorded codewords in a sequential manner, using a two-step procedure at each decoder stage. Specifically, the decoder at stage ℓ first computes soft outputs $\mathbf{z}^{(\ell)}$ corresponding to codeword $\mathbf{c}^{(\ell)}$. These soft outputs are used to calculate an estimate for $\mathbf{c}^{(\ell)}$, and are also used as *a priori* information during the next stage – stage $\ell + 1$ – of the overall decoding process. The multistage decoder architecture is shown in Fig. 3.

In [5], it was shown that an m-level system with MAP channel detectors and optimized component LDPC codes can operate at rates that converge to the SIR, as $m \to \infty$. In practice, it has been found that good performance can be obtained with relatively small values of m. For example, Fig. 4 shows results for the EPR4 channel with $m = 3$ levels. The capacity upper bound is the Gaussian-input water-filling capacity; the capacity lower bound is the mutual information rate for an optimized 8-state binary Markov input process. Also shown are the computed SIR curve and the overall system rate $R_{av,3}$. The symbols correspond to iterative-decoding thresholds for 3-level MLC/MSD systems with component LDPC codes having maximum left degree 30. For each point, its percentage of the SIR and its dB gap from the SIR threshold are shown. The parenthesized values are the percentage of $R_{av,3}$ and the dB gap from the $R_{av,3}$ threshold.

1.4. Outline of the Paper

The remainder of the paper is organized as follows. In Section 2, we discuss several bounds and simulation-based estimates of the SIR and the capacity of 2D ISI channels, highlighting the difficulties encountered in extending

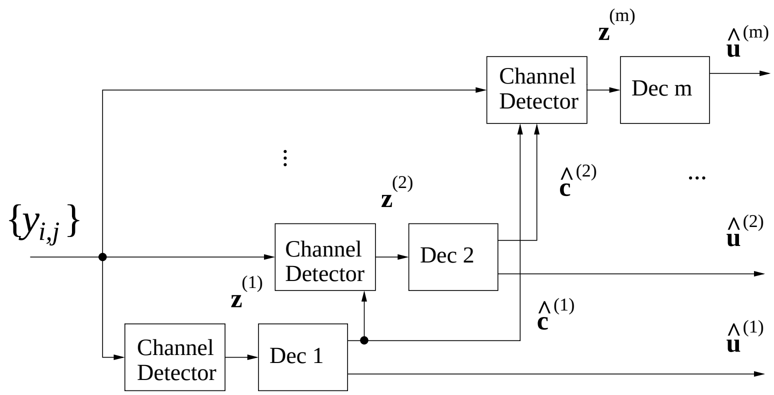

Figure 3. Multistage decoder architecture.

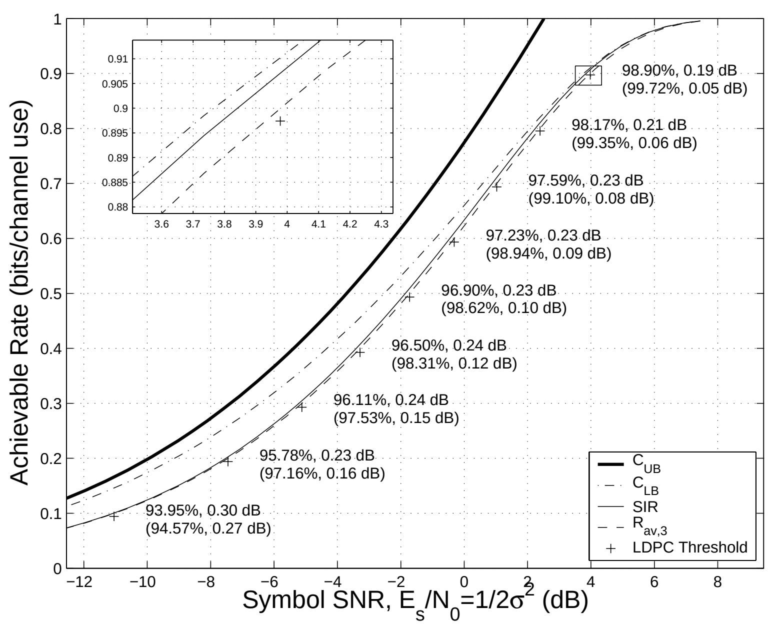

Figure 4. MLC/MSD thresholds for the EPR4 channel with $m = 3$.

to 2D the methods developed for 1D ISI channels. In Section 3, we examine the problem of optimal detection for 2D ISI channels, and consider several detection algorithms that offer various tradeoffs between performance and complexity. In Section 4, we examine coding and detection architectures that exploit the power of graph-based codes and iterative detection and decoding methods, and we compare their performance with the bounds and numerical estimates of information rates. Finally, in Section 5, we conclude with a brief discussion of directions for future research.

2. INFORMATION RATES AND CAPACITY FOR 2D ISI CHANNELS

It is widely believed that limitations on present-day track-oriented recording technologies may soon stand in the way of continued advances in storage capacity and data transrer rate. Consequently, considerable effort is being devoted to the exploration and development of multi-track and page-oriented storage techniques, such as two-dimensional optical storage (TwoDOS) [1] and optical holographic recording [2].

Two-dimensional (2D) finite-state intersymbol interference (ISI) channels serve as useful models in the study of these recording channels. The simplest discrete-time model of this kind is given by

$$y_{i,j} \quad = \quad \sum_{k=0}^{n_1-1} \sum_{l=0}^{n_2-1} h_{k,l} x_{i-k,j-l} + n_{i,j},$$

where $x_{i,j}$ is the 2D binary channel input, $y_{i,j}$ is the channel output, $h_{i,j}$ is the finite-support impulse response, and $n_{i,j}$ is i.i.d, zero-mean Gaussian noise with variance $\sigma^2 = N_0/2$.

In holographic recording, one of the simplest ISI channel models has the 2×2 impulse response

$$h_1 = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

In the TwoDOS system, bits are recorded in a spiral band of rows stacked together in a hexagonal packing. The "nearest neighbor model" for the 2D ISI in the hexagonal configuration is equivalent to 2D ISI in a rectangular configuration with 3×3 impulse response

$$h_2 = \frac{1}{\sqrt{10}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

The definition of capacity and SIR extend naturally to these 2D channels. The capacity is the supremum of the mutual information rates over all 2D channel input distributions $\mathcal{X}$

$$C = \sup_{\mathcal{X}} I(\mathcal{X};\mathcal{Y}).$$

The SIR corresponds to the case where the 2D inputs are i.i.d., equiprobable binary digits.

As in the case of 1D recording channels, the SIR and capacity of 2D channels can be interpreted as theoretical limits on achievable storage densities, and, therefore, they provide useful benchmarks against which signal processing and coding techniques can be measured. Early investigations of the capacity of 2D holographic storage devices often considered the channel as memoryless (see, for example, [13]). With the introduction of simulation-based methods for accurately computing the SIR and bounds on the capacity of 1D channels with memory, it is natural to investigate their applicability to 2D ISI channels.

The obstacle faced when trying to extend the 1D approach to two dimensions (and higher) is the lack of a simple algorithm like the BCJR forward recursion to compute the probability of a large channel output-array realization. In [14], this problem was circumvented by using conditional entropies of smaller output arrays as the basis for lower and upper bounds on the entropy rate of a large output array. For 2D channels having an impulse response with small support, the 1D Monte Carlo technique can be adapted to the computation of the conditional entropies, yielding useful lower and upper bounds on the SIR. For channels with larger impulse

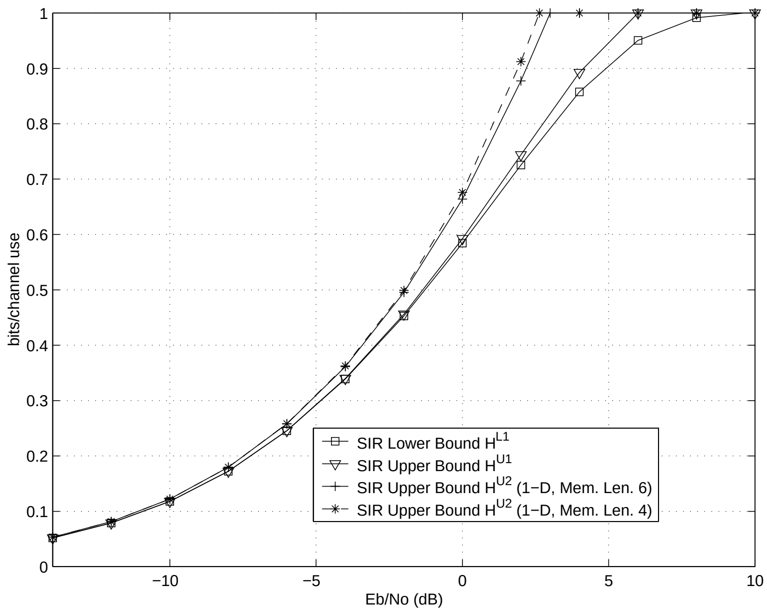

Figure 5. Bounds on the SIR of the channel h_1 with binary inputs.

response support, 1D "auxiliary channels" that capture some of the statistical dependencies of the 2D channel can be used to obtain useful bounds.

Fig. 5 shows upper and lower bounds of both types for the h_1 channel with 2×2 impulse response. Note that the looser upper bounds computed using the 1D auxiliary channel technique are nevertheless fairly close to the conditional entropy upper bound for low rates. The 1D auxiliary channel technique was also applied to the "TwoDOS channel" with 3×3 impulse response h_2. An upper bound on the SIR for this channel is shown in Fig. 6.

In [15], a more general, and often more tractable, approach to deriving bounds on the SIR of 2D channels with larger ISI span was obtained by extending the 1D bounds proposed in [16]. Numerical estimation of some of these bounds can be simplified using the Monte Carlo techniques mentioned above. For certain channels, at high SNR, these lower and upper bounds on the SIR prove to be tighter. In particular, for the h_1 channel, the computed lower and upper bounds are nearly indistinguishable.

Tightly bounding the capacity of an input-constrained 2D channel can be even more difficult. In [15], the result in [17] is extended to give an upper bound on the capacity when the channel inputs are i.i.d. and real, subject to an input energy constraint. This represents an upper bound on the capacity with binary inputs. Non-trivial lower bounds on the capacity were derived from information rate bounds for optimized input distributions of various types. The capacity upper bound and several capacity lower bounds for the h_1 channel are shown in Fig. 7. Also shown in the figure is the tight lower bound on the SIR mentioned in the previous paragraph.

Recently, another approach to estimating the SIR of 2D ISI channels was proposed [18]. It makes use of the "generalized belief propagation" (GBP) algorithm, an extension of belief propagation, that plays the role of the forward recursion of the BCJR algorithm in the 1D case [19]. Although not proven to be a bound or a precise estimate of the SIR, empirical results show good agreement with the known SIR bounds, and they suggest that the accuracy of the estimate may be validated upon further analysis [18].

3. DETECTION METHODS

We now consider the practical problem of designing equalization and detection strategies that, when combined with coding, can approach the information-theoretic limits described in the previous section.

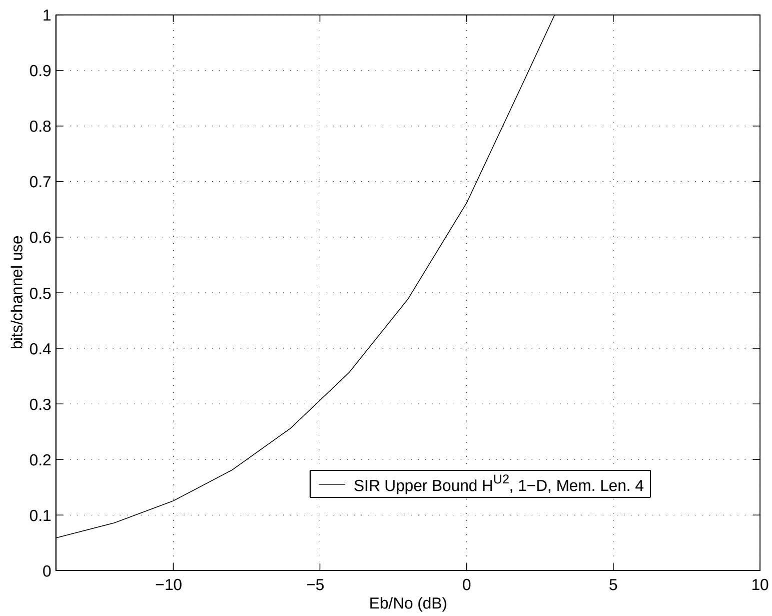

Figure 6. Upper bound on the SIR of the TwoDOS channel h_2 with binary inputs.

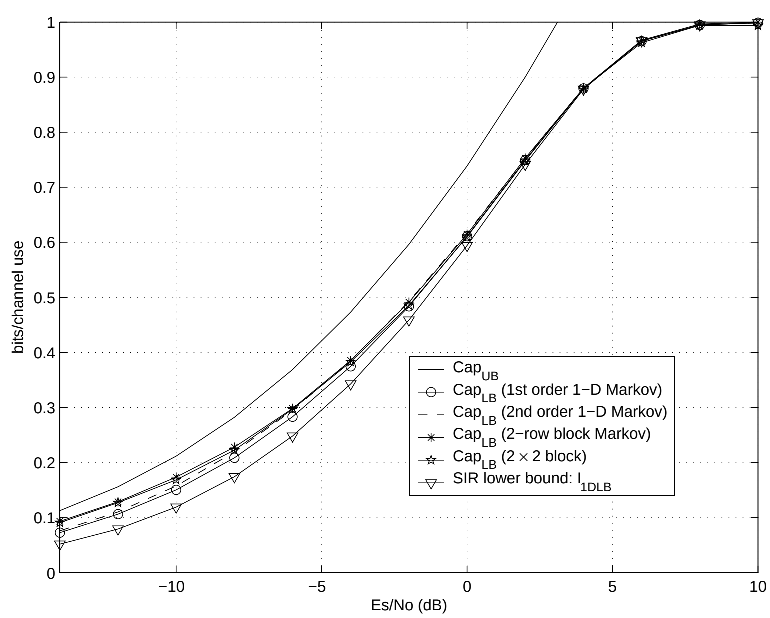

Figure 7. Bounds on the capacity of the channel h_1 with binary inputs.

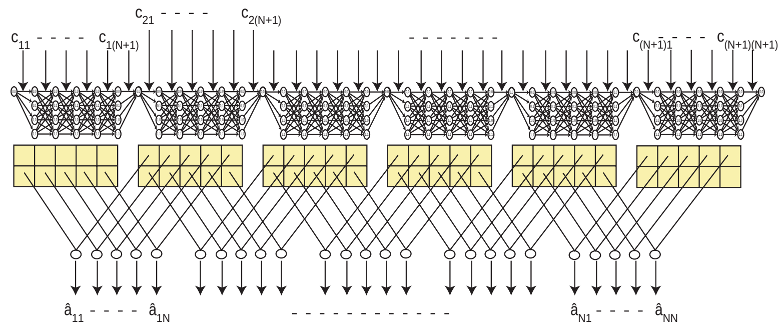

Figure 8. Graphical structure for IMS detection (2x2 channel impulse response).

3.1. BCJR-type Methods

As mentioned earlier, for 1D ISI channels, the BCJR algorithm [3] provides optimal (MAP) bit detection, and it does so with bounded complexity per bit. In the absence of a direct counterpart to the BCJR algorithm to simplify the 2D MAP detector, several suboptimal detection methods have been proposed (see, for example, [20], [21], [22], [23], [24], [25], [26], [27], [28], [29]). Many of these incorporate some form of iterative, message-passing operations as a means to reduce complexity.

For the TwoDOS application, a "strip-wise" Viterbi detection technique was proposed in [30]. The broad spiral of N tracks is viewed as a collection of partially overlapping strips, each containing a contiguous subset of, say, $L < N$ rows. Starting with the top strip, the "feed-forward" strip-wise detector sequentially applies a Viterbi-like detector to each strip, using the output of each successive strip detector to assist the detector for the next strip.

An extension of this approach is the iterative multi-strip (IMS) algorithm, introduced in [31]. Assume the channel impulse response support spans an $L \times L$ region. The IMS algorithm applies the BCJR algorithm to a trellis that reflects the dependence of each row in the channel output array upon a finite span of L rows ("multi-strip") in the input array. It generates log *a posteriori* probability ratios (LAPPR) for the pertinent input rows, given the output row. Every input row (ignoring rows near the edges of the array) belongs to L multi-strips, so LAPPR values for the row are generated by L multi-strip BCJR detectors. A message-passing rule is used to share these values among the L relevant multi-strip detectors for use in the next iteration of multi-strip detection. Fig. 8 depicts a graphical structure for IMS detection of a 2D ISI channel with 2x2 impulse response. (It should be pointed out that several versions of the IMS algorithm, incorporating reduced complexity alternatives to the BCJR multi-strip detector, have been evaluated in [32].)

In [31], the IMS algorithm was applied to the 3×3 TwoDOS channel. Its performance as a function of the number of iterations was compared to a truncated union bound estimate of a 2D ML detector. The union bounds included contributions from 2D error events corresponding to the first few terms of the distance spectrum, as defined by Chugg [20]. After 10 iterations, the IMS performance was within 0.5dB of the bound.

The GBP algorithm, mentioned above, represents, in a sense, a generalization of the IMS algorithm. It decomposes the output array into possibly overlapping regions, within which optimal bit detection is performed, and between which soft-output messages are exchanged iteratively. Computer simulations on small arrays for which full MAP detection can be applied indicate that the IMS algorithm and, more generally, the GBP algorithm with appropriately selected regions, can provide near-optimal detection [19].

3.2. Parallel Message-Passing Detection

A graph-based detection approach similar in spirit to the MPPR detector can be used for two-dimensional ISI channels. This message-passing algorithm uses a bipartite graphical representation of the 2D ISI channel, in which interconnections between an array of channel input nodes and a layer of "output function" nodes capture the dependence of the outputs upon the inputs. An advantage of this approach is its ability to incorporate nonlinear characteristics of the 2D ISI channel. See [33], [24] for a detailed discussion.

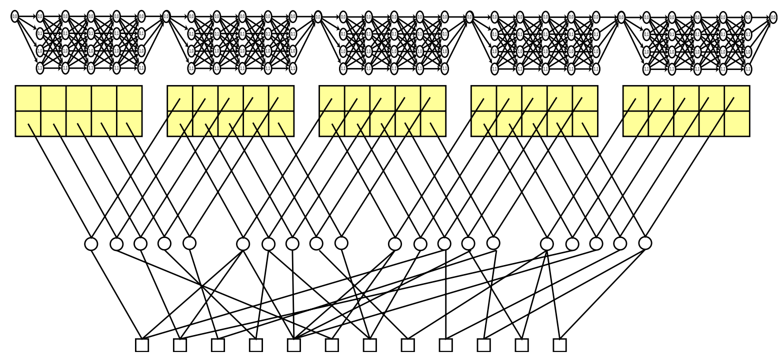

Figure 9. Decoding graph for joint IMS detection and message-passing LDPC decoding.

4. COMBINED CODING AND DETECTION

In order to approach achievable information rates, we must incorporate coding into the system. This section describes several approaches, similar in nature to those used in the 1D case.

4.1. Concatenated Architectures with Iterative Decoding

Recall that, for 1D channels, the BCJR (or MPPR) detector was coupled with the Tanner graph of an LDPC code, yielding a compound graph upon which a suitably defined message-passing procedure could be used quite effectively for joint equalization and decoding.

Similarly, iterative joint equalization and decoding of LDPC codes can be implemented for 2D ISI channels by combining a 2D channel detection graph with the Tanner graph describing an LDPC code applied to the 2D array. Fig. 9 shows a graphical structure for an IMS detector concatenated with an LPDC decoder.

In [34], joint equalization and decoding of the LDPC-coded nonlinear Two-DOS channel was implemented by means of a message-passing algorithm on a factor graph representing the entire system. This "full-graph" contains three "layers" corresponding to the check nodes of the LDPC code, variable nodes for the LDPC code bits that are input to the channel, and channel output function nodes corresponding to the observed outputs. In addition, modified density evolution techniques similar to those developed in [9] were used to compute noise tolerance thresholds for the full graph algorithm applied to regular LDPC codes. These threshold values, although not yet shown to be true asymptotic thresholds, were found to empirically reflect code performance for large blocksize. It would be interesting to compare these computed thresholds, as well as the empirically observed thresholds of the full graph decoder, to the achievable rates of the TwoDOS ISI channel.

4.2. Multistage Decoding Architectures

The MSD architecture can be easily extended to 2D channels [35]. After encoding, however, the m codewords must be interleaved in an appropriate fashion to form a 2D channel input array. An example of an interleaver pattern for $m = 3$ encoders into an array of 30 columns is shown in Fig. 10.

The IMS algorithm is used in place of an optimal detector, and can be readily incorporated into the MSD architecture. By simulating the operation of the MSD, one can generate outputs for each decoder stage. From these, a histogram representing the marginal conditional output distribution can be estimated. These approximate distributions can be used to estimate achievable code rates $R_m^{(\ell)}$ at each system level.

Fig. 11 shows achievable rates for the h_1 channel model with IMS decoding at each stage of an m-level system, for $m = 1, 2, 3$. Also shown are upper and lower bounds on the channel SIR. It can be seen that a 3-level system nearly achieves the SIR lower bound. Also shown are computed thresholds for LDPC codes optimized using the simulated histograms at each stage.

The MLC-MSD architecture is amenable to other coding and detection methods, as well. In [35], [36], zero-forcing equalization (ZFE) and linear noise prediction were substituted for the more complex IMS detector, and

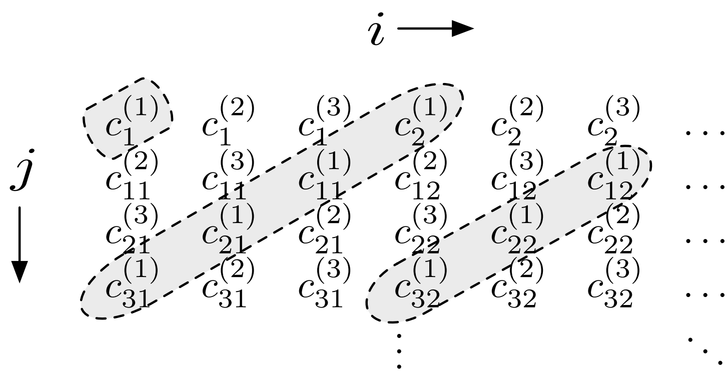

Figure 10. Example of 2D interleaving pattern for $m = 3$.

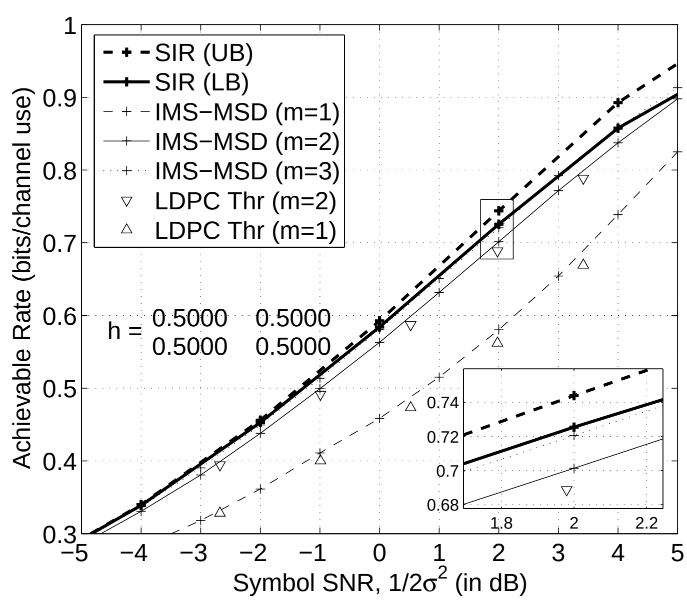

Figure 11. SIR bounds, achievable rates, and LDPC code thresholds for channel h_1 with binary inputs.

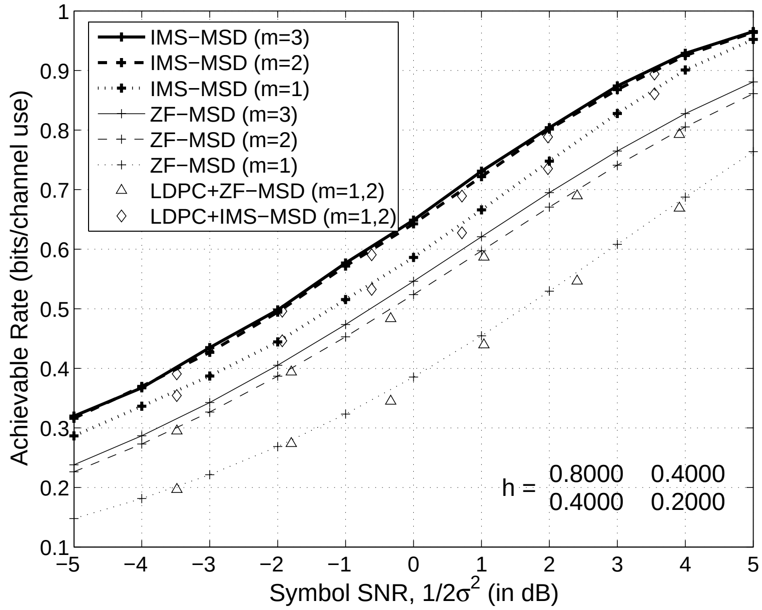

Figure 12. Comparison of IMS-MSD and ZFE-MSD performance (with and without LDPC codes) for channel h_3 with binary inputs.

closed form expressions for achievable rates were derived. For the channels investigated, the performance results, both with and without LDPC codes, are substantially inferior to those obtained with the IMS-based system. This can be seen in Fig. 12, where both methods were applied to the 2D channel with impulse response

$$h_3 = \begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}.$$

5. CONCLUDING REMARKS

This paper has reviewed some recent results on achievable information rates of 2D ISI channels, and has compared the performance of various coding and detection techniques to these theoretical limits. Although some progress has been made, many research challenges remain with regard to the information-theoretic analysis of 2D channels and the design and performance evaluation of practical and efficient equalization, detection, and coding schemes.

For example, we have primarily considered the symmetric information rate of 2D channels characterized by linear, stationary ISI with relatively small impulse response support. There is a need for improved, computable bounds for 2D channels with more severe, possibly nonlinear, and possibly non-stationary ISI.

The problem of designing coding and detection schemes for more general models of 2D ISI channels also warrants further attention. For example, it would be interesting to generalize to 2D ISI channels the techniques in [37], [38], where LDPC codes and irregular-repeat-accumulate (IRA) codes have been optimized for memoryless holographic recording channels with spatially-varying SNR.

Some of the results presented here indicate that, in order to approach capacity at low SNR, linear (coset) codes, such as LDPC codes, will not suffice, and an additional shaping code must be introduced to surpass the SIR. The incorporation of such codes into system architectures for 1D ISI channels has been investigated in [39], [40], but, even in the 1D case, the design of practical coding and detection schemes that approach the capacity remains an open problem.

It is clear that, in the future, the push to develop two-dimensional optical data storage technologies will continue to provide a driving force behind all of these lines of research.

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